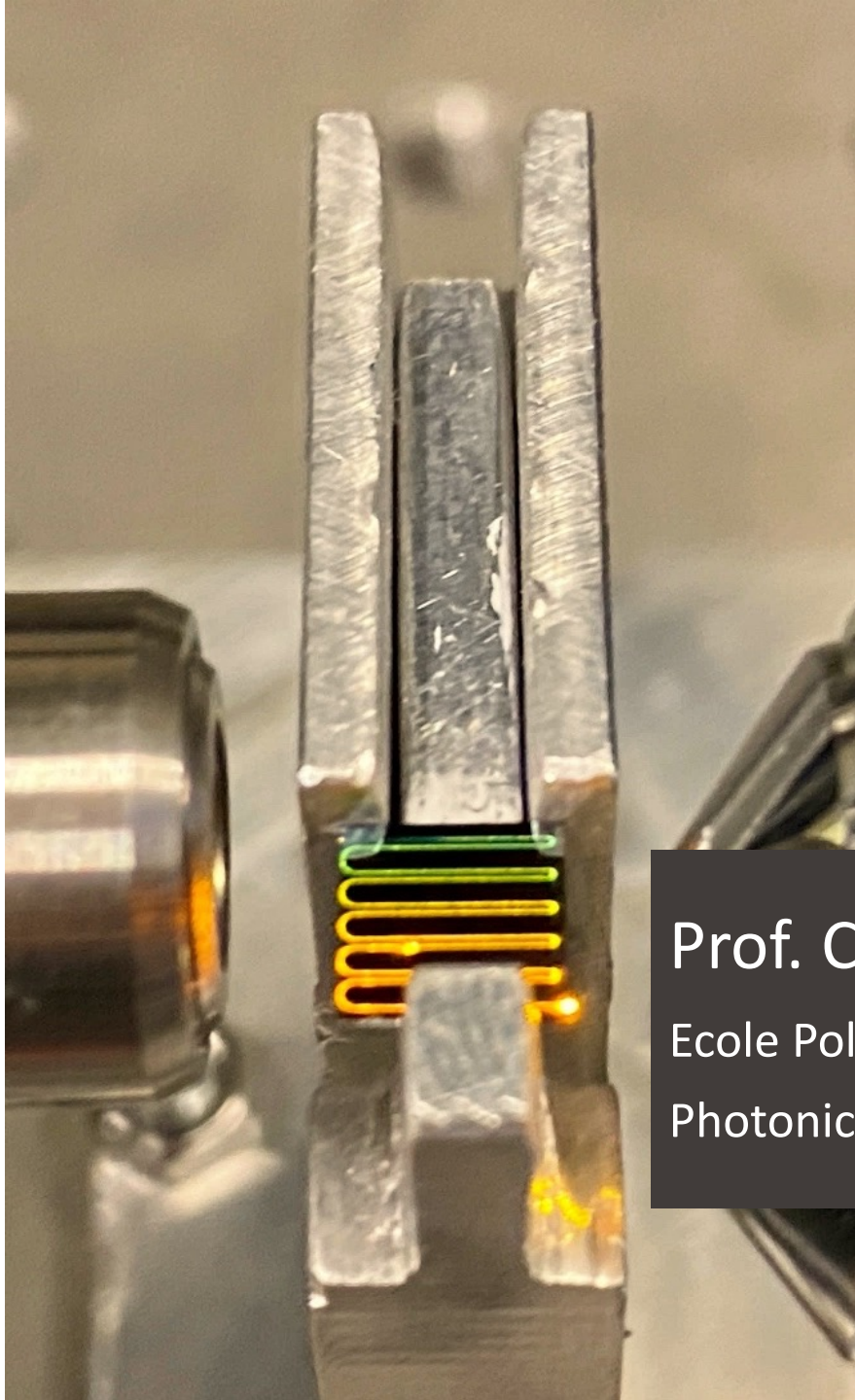




Integrated nonlinear optics

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SMYLE Summerschool Neuchâtel, Switzerland

Integrated optics / photonics

Integration of optical functions on a circuit to overcome fundamental issues of free space
alternative: size, weight, stability, alignment, cost ...

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ASPECTS OF ELECTRICAL COMMUNICATION

Volume 48 September 1969 Number 7

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Integrated Optics: An Introduction

By STEWART E. MILLER

(Manuscript received January 29, 1969)

IEEE JOURNAL OF QUANTUM ELECTRONICS, VOL. QE-22, NO. 6, JUNE 1986

All-Silicon Active and Passive Guided-Wave Components for $\lambda = 1.3$ and $1.6 \mu\text{m}$

RICHARD A. SOREF, SENIOR MEMBER, IEEE, AND JOSEPH P. LORENZO

JOURNAL OF LIGHTWAVE TECHNOLOGY, VOL. LT-5, NO. 9, SEPTEMBER 1987

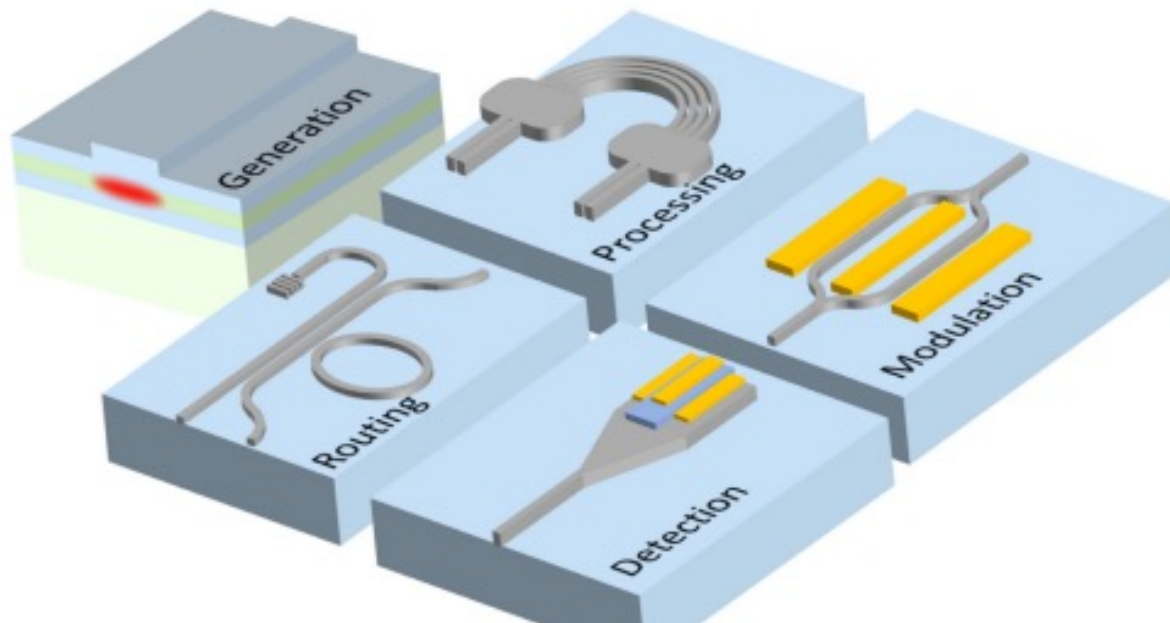
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Monolithic Integration of InGaAsP/InP Distributed Feedback Laser and Electroabsorption Modulator by Vapor Phase Epitaxy

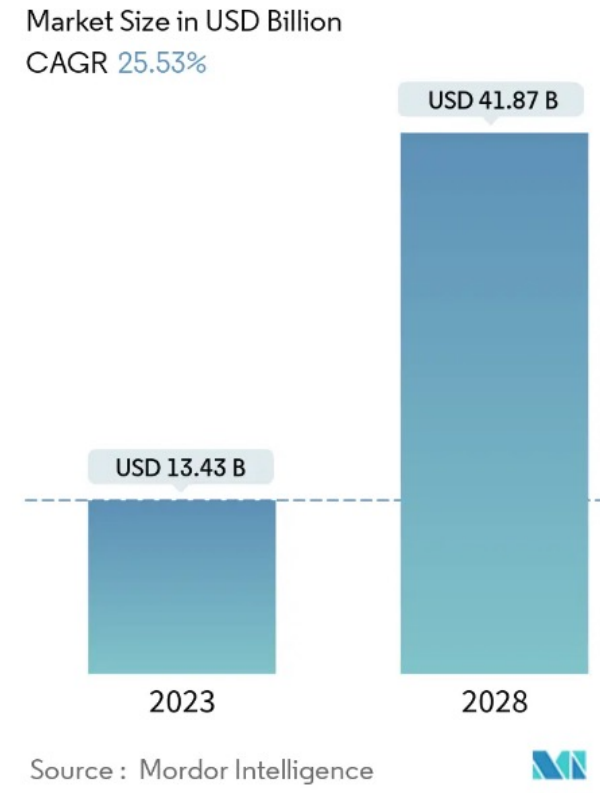
MASATOSHI SUZUKI, MEMBER, IEEE, YUKIO NODA, HIDEAKI TANAKA, SHIGEYUKI AKIBA,
YUKITOSHI KUSHIRO, AND HIDEO ISSHIKI

Photonic Integrated Circuit (PIC)

- Photonics circuit: integrated optical circuit where it is possible to combine two or more passive and active devices (building blocks) to generate and manipulate the optical signal.



Siew, Shawn Yohanes, et al., Journal of Lightwave Technology 39.13 (2021): 4374-4389.



Nonlinear optics

Study of light-matter interaction when the optical field is strong enough to change material properties

- Microscopic scale: light field hitting atoms results in displacement of electrons and induced dipoles
- Macroscopic scale: nonlinear dependence of the material polarization on the applied field.

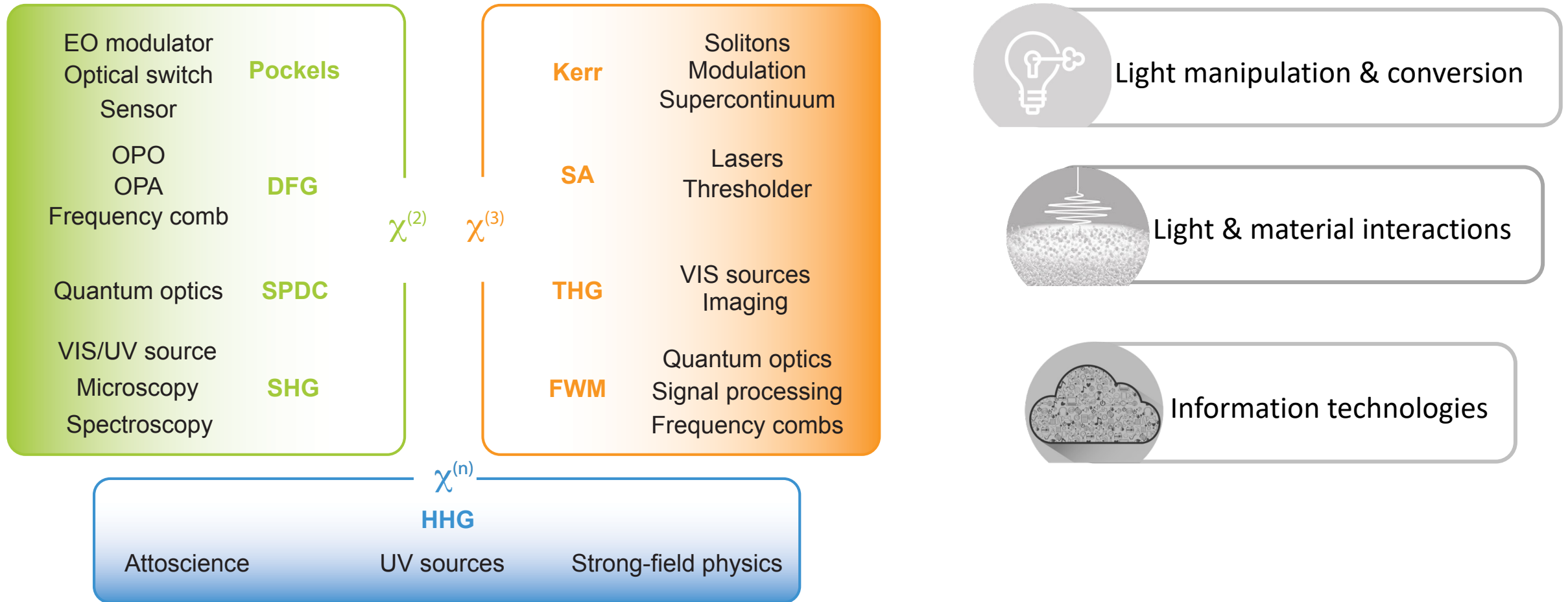
$$\vec{P} = \varepsilon_0(\chi^{(1)}\vec{E} + \varepsilon_0\chi^{(2)}\vec{E}^2 + \varepsilon_0\chi^{(3)}\vec{E}^3)$$



$$\omega_1 + \omega_1 = 2\omega_1$$

$$1064 \text{ nm} \rightarrow 532 \text{ nm}$$

Optical nonlinear effects and applications



There is a need for integrated nonlinear building blocks to bring new functionalities to the chip-scale

In this lecture

Quick basic of integrated optics and intro to nonlinear optics in waveguides

- Theory of optical waveguide
- Important parameters for nonlinear optics
- Platforms of interest

Integrated nonlinear optics in waveguides – illustration with supercontinuum generation

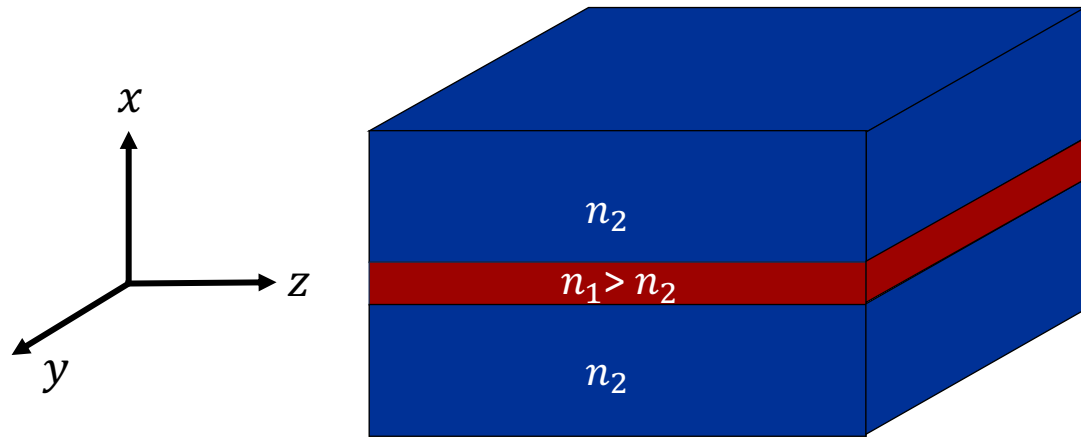
- Pulse propagation physics
- Design rules
- Demonstrations

Integrated nonlinear optics in microring resonators

- Basics of microresonator
- Nonlinear behavior
- Illustration of Kerr comb generation and dissipative solitons

Theory of optical waveguide

First let's consider the case of a simple slab (or planar) waveguide



- Slab of dielectric (refractive index n_1) between two semi-infinite regions (refractive index n_2)
 - Waveguide assumed infinitely wide in y and z directions
 - Invariance of translation along y and z

$$\underline{\mathbf{E}}(r) = \mathbf{E}(x) \exp(-j\beta z)$$

$$\underline{\mathbf{H}}(r) = \mathbf{H}(x) \exp(-j\beta z)$$

- β the propagation constant of the guided mode

TE and TM modes

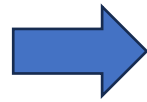
Maxwell's equations

$$\nabla \times \underline{\mathbf{E}}(r) = -j\omega\mu_0 \underline{\mathbf{E}}(r)$$

$$\nabla \times \underline{\mathbf{H}}(r) = j\omega\varepsilon \underline{\mathbf{R}}(r)$$

+

Invariance of translation of
the structure



$$E_y = -\frac{\omega\mu_0}{\beta} H_x$$

$$H_y = \frac{\omega\varepsilon}{\beta} E_x$$

$$\frac{\partial E_y}{\partial x} = -j\omega\mu_0 H_z$$

$$\frac{\partial H_y}{\partial x} = j\omega\varepsilon E_z$$

$$-j\beta H_x - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y$$

$$-j\beta E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu_0 H_y$$

2 sets of totally independent equations, 2 kinds of solutions:

- **Transverse electric (TE):** (E_y, H_x, H_z)
- **Transverse magnetic (TM):** (H_y, E_x, E_z)

Theory of optical waveguide

- Common approach is to solve the *wave equations*, which reduce to the Helmholtz equation

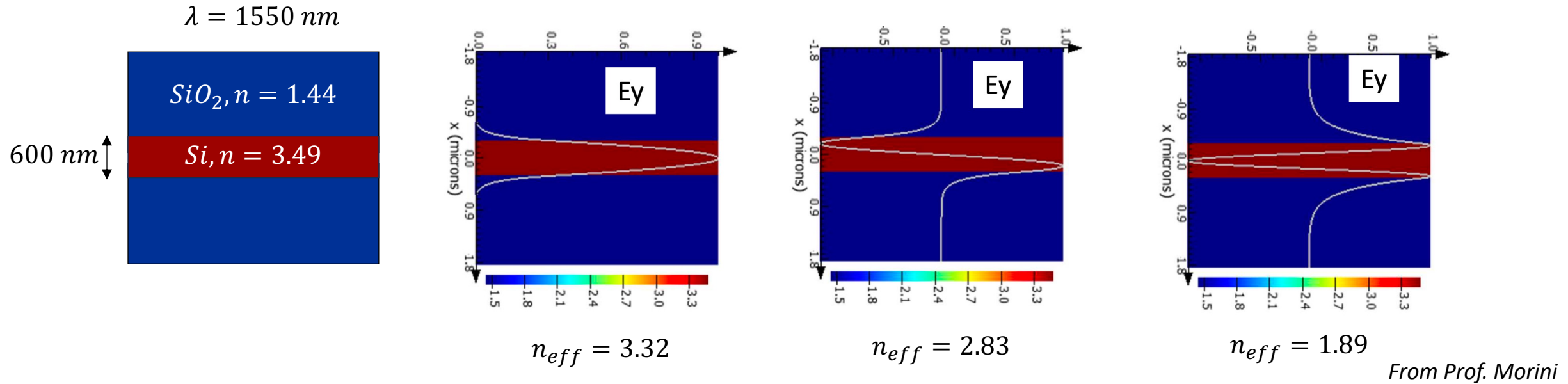
$$\nabla^2 U + k^2 U = 0 \quad \text{with } k = nk_0 = n \frac{\omega}{c_0}$$

- In slab, for TE modes the only non-vanishing electric field component is E_y :

$$\frac{\partial^2 E_y}{\partial x^2} + (k^2 - \beta^2) E_y = 0$$

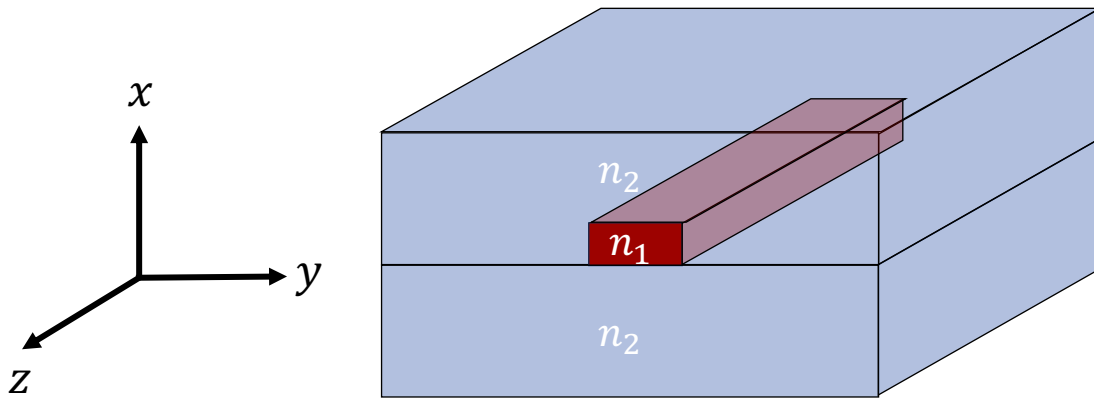
- Combined with *boundary conditions* (interface of 2 mediums, continuity relations)
- Guided modes have $n_2 k_0 < \beta < n_1 k_0 \rightarrow n_2 < n_{eff} < n_1$

Illustration of mode calculations



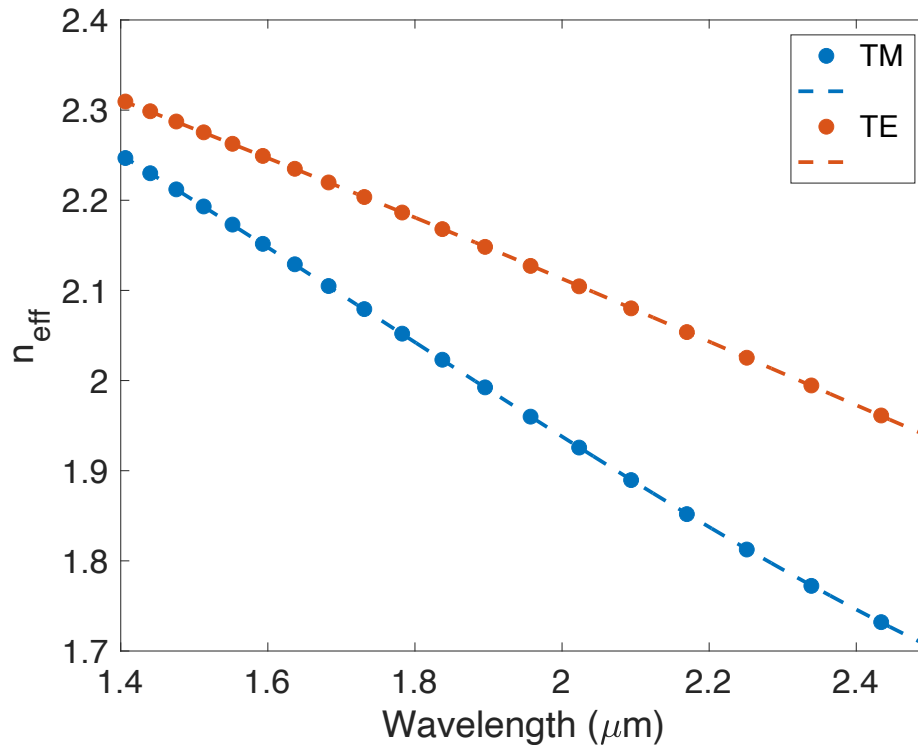
- The number of guided modes depends on: the refractive indexes, slab thickness, wavelength.
 - Increases with increasing thickness
 - Increases with decreasing wavelength
- Light confinement in the core depends on dimensions, wavelength and refractive index
 - Confinement decreases with increasing wavelength
 - Tighter light confinement with large index contrast structures

Optical waveguide



- The waveguide is no longer infinite in the y direction
 - Have quasi TE and quasi TM modes
- Most of the properties of guided modes in slab waveguides can be translated to 2D waveguides
- Mode calculations can be done using FDE (Ansys/Lumerical) or FEM (Comsol)

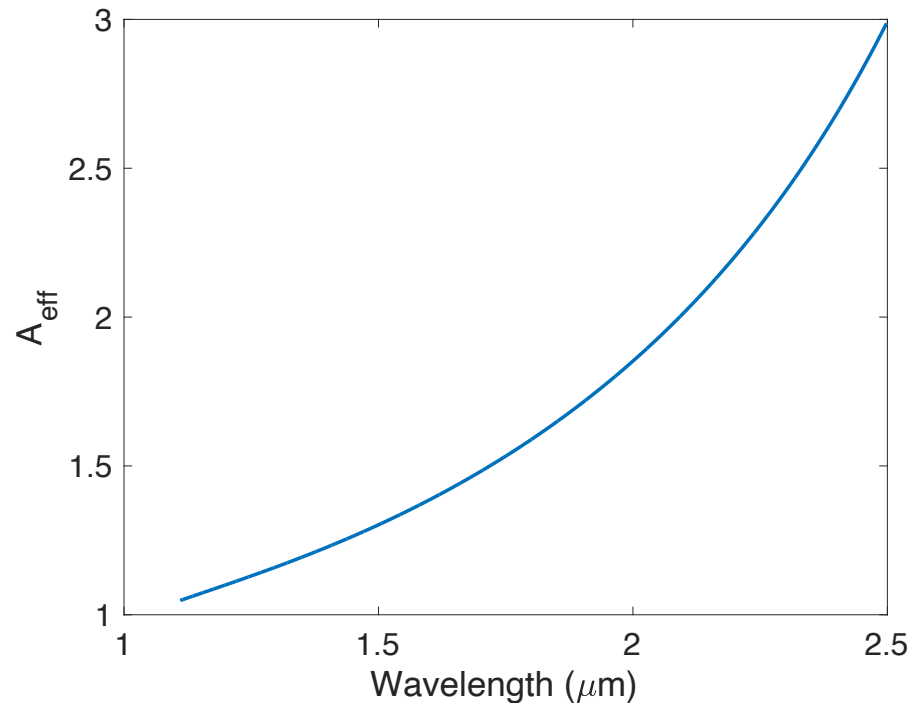
Influence of wavelength in waveguide - dispersion



- SiC waveguide, 500 nm x 1200 nm cross section SiO₂ cladding
- Effective index varies with wavelength
- Varies nonlinearly with frequency

$$\beta(\omega) = \beta_0 + \beta_1(\omega - \omega_0) + \frac{\beta_2}{2!}(\omega - \omega_0)^2 + \dots$$

Effective mode area - confinement



- Introduced naturally during the derivation of the nonlinear Schrodinger equation

$$A_{\text{eff}} = \frac{\left(\iint_{-\infty}^{\infty} |\mathbf{F}(x, y)|^2 dx dy \right)^2}{\iint_{\sigma_3} |\mathbf{F}(x, y)|^4 dx dy}$$

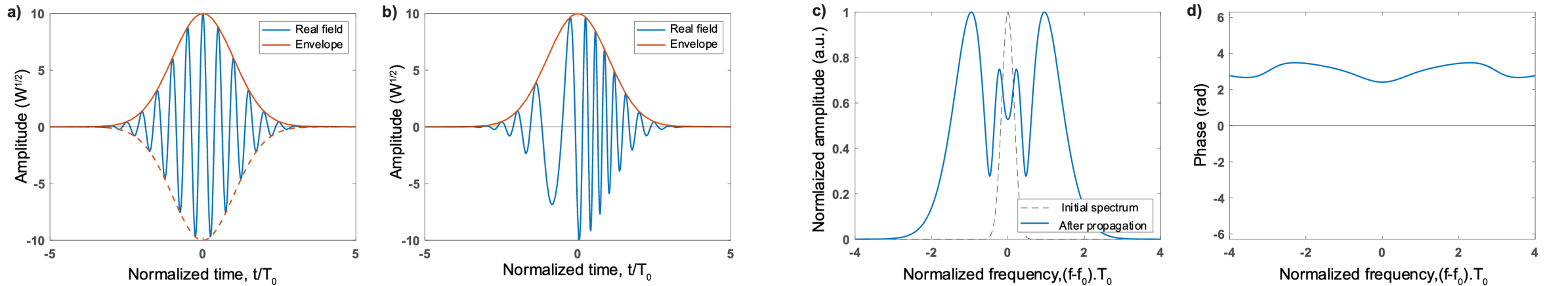
- Used to quantify confinement and of great importance for nonlinear optics

Why integrated nonlinear optics?

- Let's take the example of light propagation in a nonlinear waveguide with 3rd order nonlinearity $\chi^{(3)}$
- Efficiency of the nonlinear process is described by the **nonlinear coefficient** $\gamma = \frac{\omega_0 n_2}{c_0 A_{eff}}$
 - With A_{eff} the effective area (waveguide property) $A_{eff} = \frac{(\iint_{-\infty}^{\infty} |\mathbf{F}(x, y)|^2 dx dy)^2}{\iint_{\sigma_3} |\mathbf{F}(x, y)|^4 dx dy}$
 - With n_2 the nonlinear refractive index (material property) $n_2 = \frac{3\chi^{(3)}}{4\varepsilon_0 c_0 n_0^2}$
- **Dispersion also plays a major role** in nonlinear processes
- Depending on the process, **loss can also be a factor to consider** (for example in microresonators)

Important parameter - nonlinearity

- Nonlinearity often quantified through the nonlinear parameter $\gamma = \frac{\omega_0 n_2}{c_0 A_{\text{eff}}}$
- Self-phase modulation (SPM) comes from the intensity dependence of n : $n(\omega, I) = n_0(\omega) + n_2 I$
 - Nonlinear phase shift is given by $\varphi_{NL} = \gamma P_0 L_{\text{eff}}$
- Characteristic length of nonlinearity: $L_{NL} = \frac{1}{\gamma P_0}$



Simulations, nonlinear propagation: $L = 10L_{NL}$, $\gamma = 2 \text{ W}^{-1}\text{m}^{-1}$, $P_0 = 100 \text{ W}$, $T_0 = 1 \text{ ps}$

SPM generated frequency components broaden the pulse spectrum, results in up chirp

Important parameter - dispersion

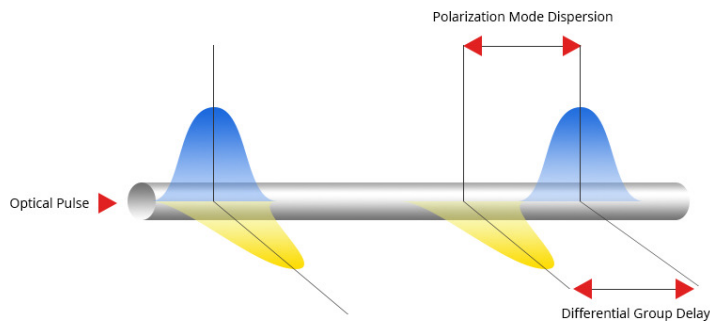
Dispersion of light

Intermodal

Modal

Polarization

Contribute in the case of multimode transmission



Intramodal

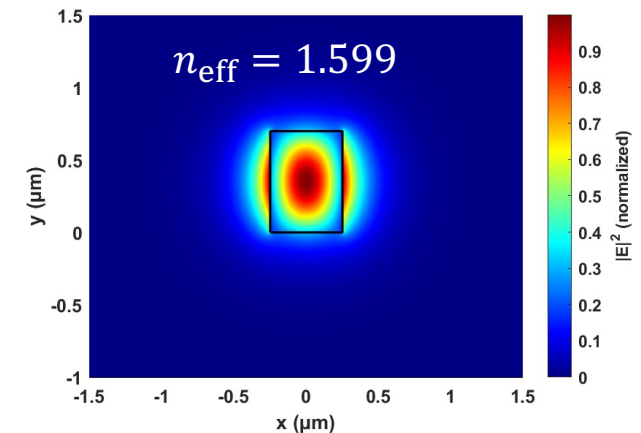
Material

Frequency dependence of bulk material refractive index material property

$$n(\lambda) = \sqrt{1 + \sum_j \frac{B_j \lambda^2}{\lambda^2 - C_j}}$$

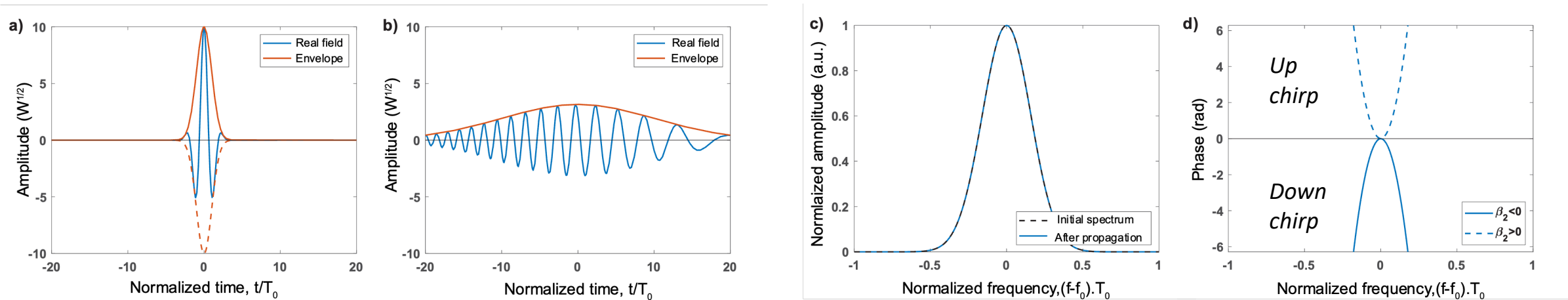
Waveguide

Geometric contribution due to distinct core/cladding material



Important parameter - dispersion

- Group velocity dispersion (GVD), $\beta_2 = \frac{\partial^2 \beta}{\partial \omega^2}$ is a key parameter (interchangeable with $D = -\frac{2\pi c_0}{\lambda^2} \beta_2$)
- Characteristic length of GVD: $L_D = \frac{T_0^2}{|\beta_2|}$



Simulations, linear propagation: $L = 10L_D$, $\beta_2 = -1 \cdot 10^{-25} \text{ s}^2/\text{m}$, $P_0 = 100 \text{ W}$, $T_0 = 1 \text{ ps}$

The 2 dispersion regimes result in pulse temporal broadening, give rise to chirping of opposite signs

Important parameter - losses

Nonlinear

- Two-photon absorption (TPA)
- Three-photon absorption (3PA)

Alleviated by right choice of material for given operating λ

Coupling

- Impact overall requirements
- Damage and stability

Can reflect technology maturity and challenges

Propagation

- Absorption and scattering: α
- Characteristic length:

$$L_{eff} = \frac{1 - \exp(-\alpha L)}{\alpha}$$

A lossy waveguide behaves like a loss-less waveguide with the same γ but with a shorter length

Some information about available material platforms

Material	Trans. Window (μm)	Bandgap	n	n_2 (cm^2/W)
SiO ₂	0.13 – 3.5	9 eV	1.46	$\sim 2.7 \cdot 10^{-16}$
Hydex	0.13 – 3.5	9 eV	1.7	$\sim 1.1 \cdot 10^{-15}$
Si	1.1 – 9	1.12 eV	3.48	$\sim 6 \cdot 10^{-14}$
Si ₃ N ₄	0.35 – 7	5 eV	2	$\sim 2.4 \cdot 10^{-15}$
SiGe (40% Ge)	1.5 – 11	~ 1.1 eV	3.6 (at 4 μm)	$\sim 4 \cdot 10^{-14}$ (at 4 μm)
Ge	2 – 14	0.7 eV	4 (at 4 μm)	$0.5 - 1 \cdot 10^{-13}$ (at 4 μm)
AlGaAs	0.57/0.87 – >6.5	1.42 – 2.16 eV	2.86 -3.5	$\sim 10^{-13}$
AlN	0.2 – >5.5	6.2 eV	2.21 (o), 2.26 (e)	$\sim 2.3 \cdot 10^{-15}$
InGaP	0.65 – MIR	1.9 eV	3.13	$\sim 10^{-13}$
Chalcogenides	Sulphides: VIS – 11	~ 2.5 eV	2 - 3	$9 \cdot 10^{-12} - 9 \cdot 10^{-14}$
	Selenide: NIR – 15	~ 1.8 eV		
	Tellurides: NIR – 20	~ 1.5 eV		
TFLN	0.4 – 5	~ 4 eV	2.21 (o), 2.13 (e)	$\sim 1.8 \cdot 10^{-15}$
SiC	0.5 – MIR	2.36 – 3.23 eV	2.6	$\sim 1 \cdot 10^{-14}$
Diamond	0.22 – MIR	5.5 eV	2.38	$\sim 0.8 \cdot 10^{-15}$
Ta ₂ O ₅	0.3 – 8	3.8 eV	2	$\sim 2 \cdot 10^{-14}$
TiO ₂	0.4 – 5.9	3.4 eV	2.4	$\sim 2.4 \cdot 10^{-14}$
TeO ₂	0.4 – 5	3.8 eV	2.1	$\sim 1.3 \cdot 10^{-14}$

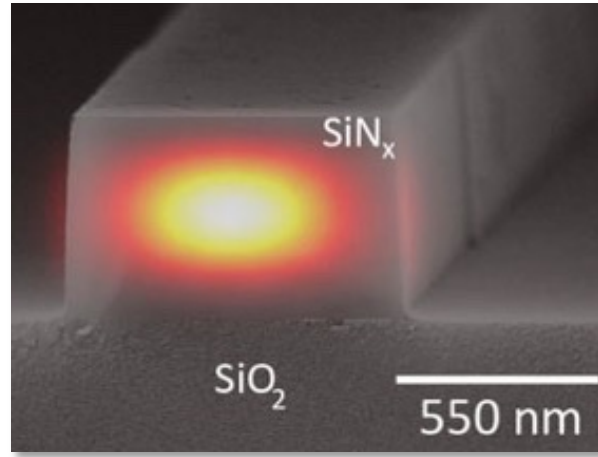
Rational for integration – *nonlinearity* (n_2) & *transparency*

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TeO ₂	0.4 – 5	3.8 eV	2.1	$\sim 1.3 \cdot 10^{-14}$

- Large library of photonic integrated platforms
- Large transparency
- Large nonlinearity

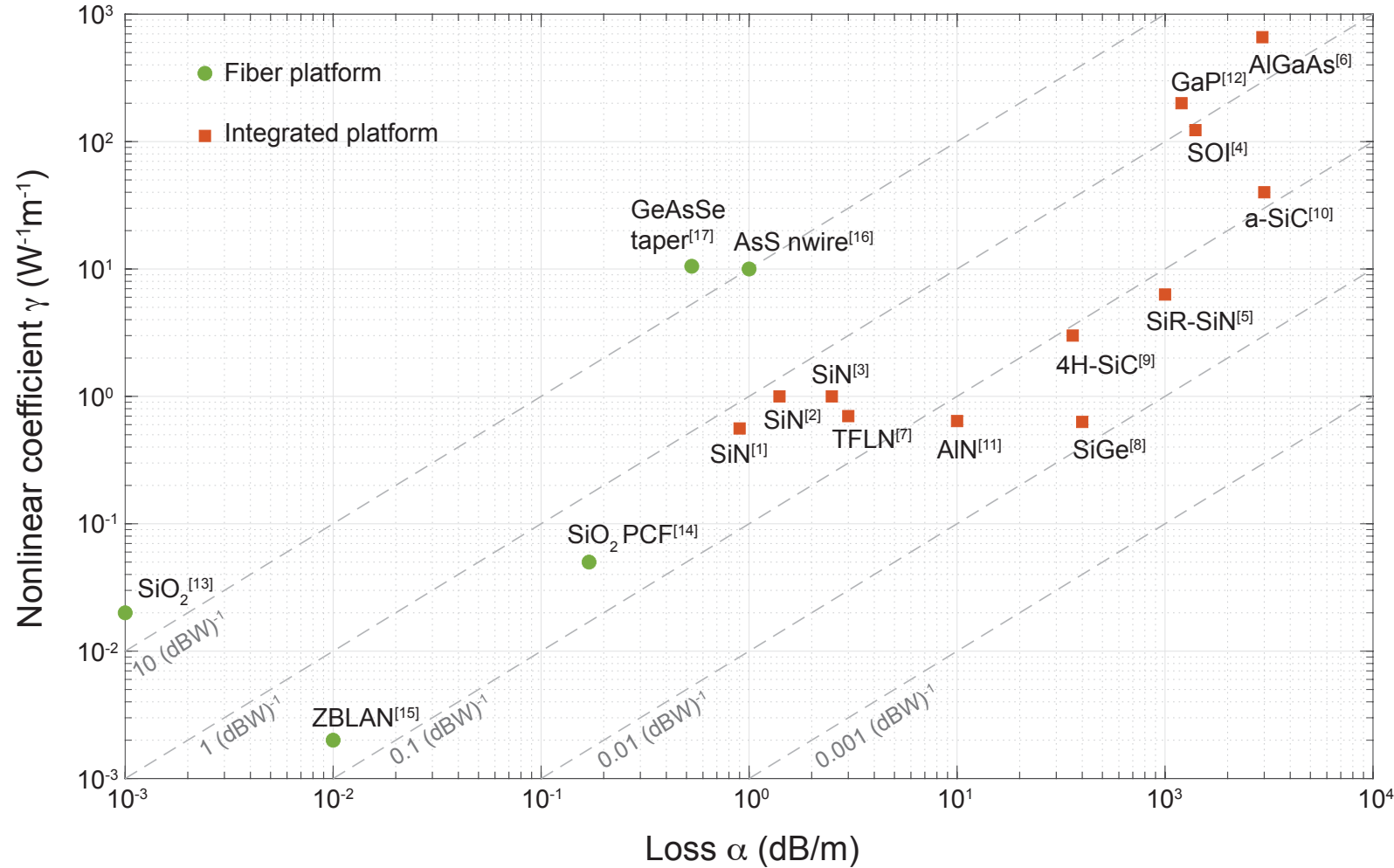
Rational for integration – *intensity* (A_{eff})

Material	Trans. Window (μm)	Bandgap	n_0	n_2 (cm^2/W)
SiO ₂	0.13 – 3.5	9 eV	1.46	$\sim 2.7 \cdot 10^{-16}$
Hydex	0.		1.7	$\sim 1.1 \cdot 10^{-15}$
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InGaP	0.1		3.13	$\sim 10^{-13}$
Sulphi				
Chalcogenides			2 - 3	$9 \cdot 10^{-12} - 9 \cdot 10^{-14}$
Seleni				
Telluri				
TFLN			2.21 (o), 2.13 (e)	$\sim 1.8 \cdot 10^{-15}$
SiC	0.		2.6	$\sim 1 \cdot 10^{-14}$
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TeO ₂	0.4 – 5	3.8 eV	2.1	$\sim 1.3 \cdot 10^{-14}$



- Possible large index difference between core and cladding
- Small guiding structures
- Higher optical field confinement

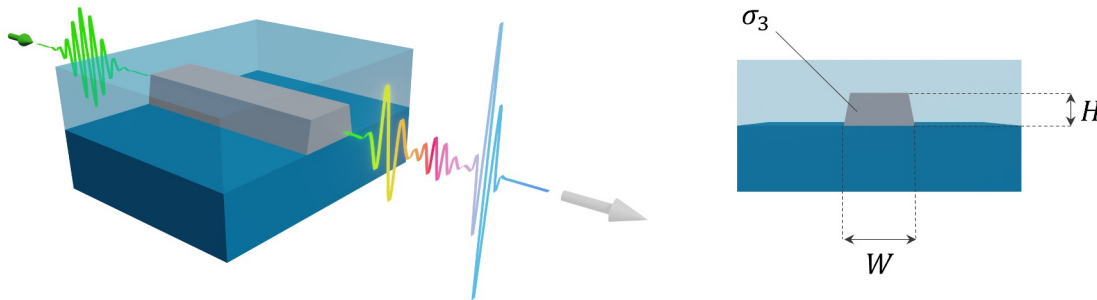
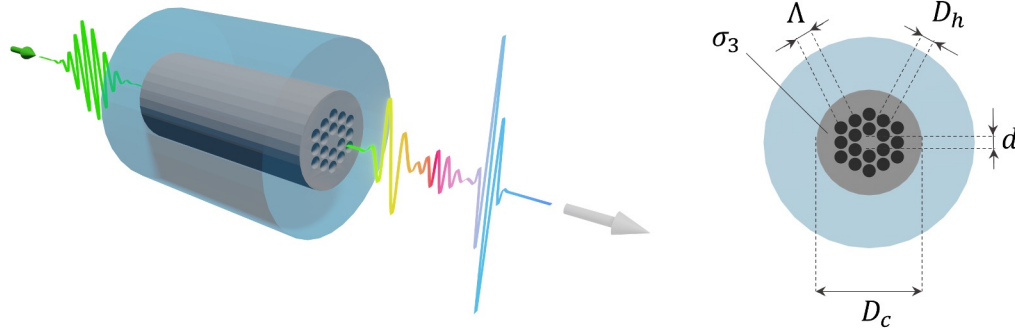
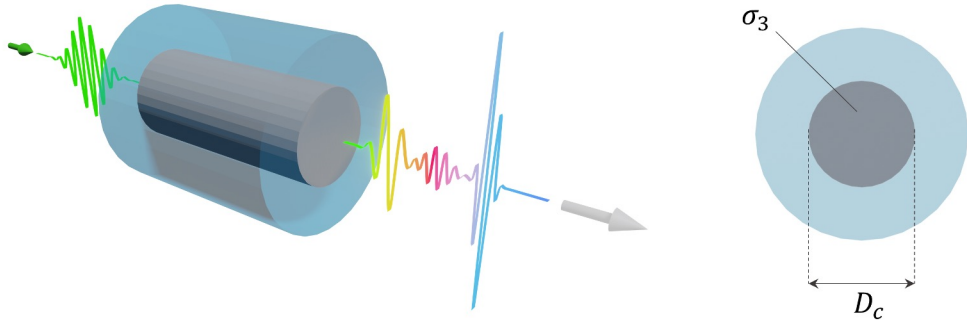
3rd order nonlinearity - figure of merit



$$FOM = \frac{\gamma}{\alpha} [dB^{-1}W^{-1}]$$

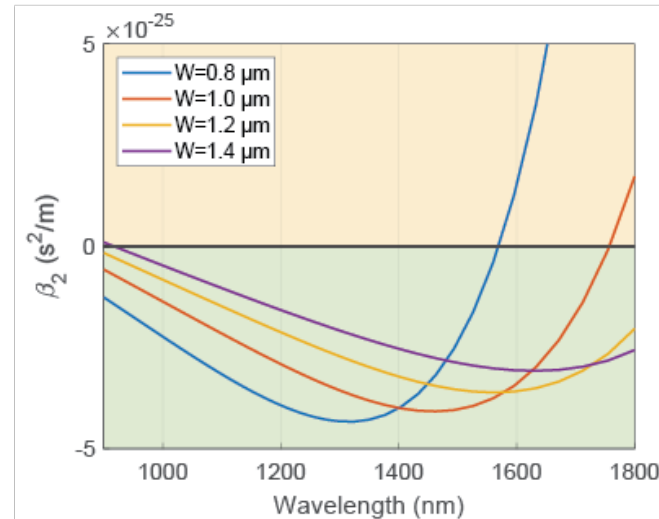
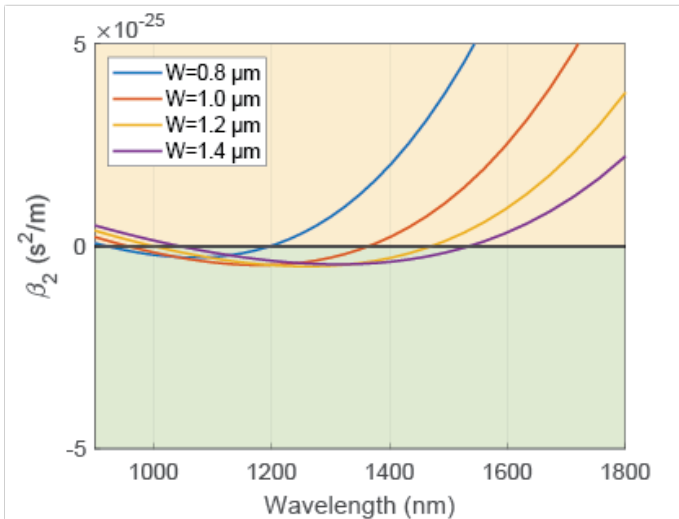
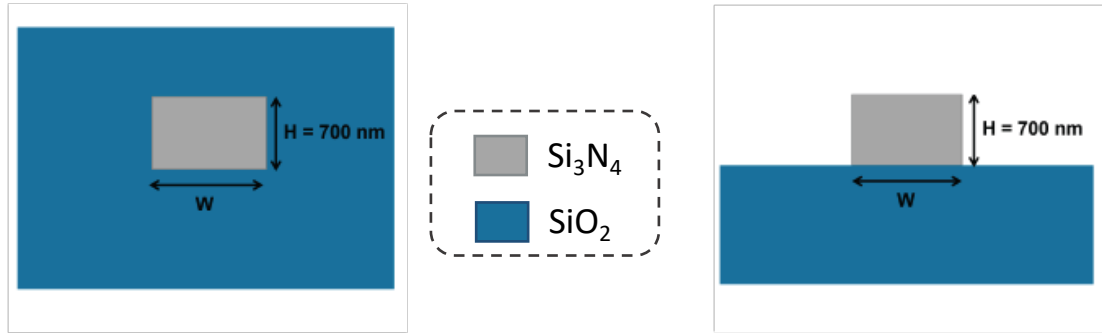
- [1] X. Ji *et al.*, **Optica** 4(6) 2017
- [2] Z. Ye *et al.*, **Science Advances** 7(38) 2021
- [3] J. Riemensberger *et al.*, **Nature** 612, 2022
- [4] M.A. Foster *et al.*, **Nature** 441(7096) 2006
- [5] C.J. Krückel *et al.*, **Opt. Express** 23(20) 2015
- [6] M. Pu *et al.*, **Optica** 3(8) 2016
- [7] M. Yu *et al.*, **Opt. Lett.** 44(5) 2019
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- [9] M.A. Guidry *et al.*, **Optica** 7(9) 2020
- [10] P. Xing *et al.*, **ACS Photonics** 6(5), 2019
- [11] K. Kiu *et al.*, **Opt. Express** 47(17), 2021
- [12] D. Wilson *et al.*, **Nature Photonics** 14, 2019
- [13] M. Hirano *et al.*, **IEEE JSTQE** 15(1) 2009
- [14] D. Aydin *et al.*, **Photonics Ad. Cong.** 2016
- [15] X. Jiang *et al.*, **Nature Photonics** 9, 2015
- [16] N. Abdukerim *et al.*, **Opt. Lett.** 41(18), 2016
- [17] S. Xing *et al.*, **Optica** 4(6) 2017

Rational for integration - *dispersion*



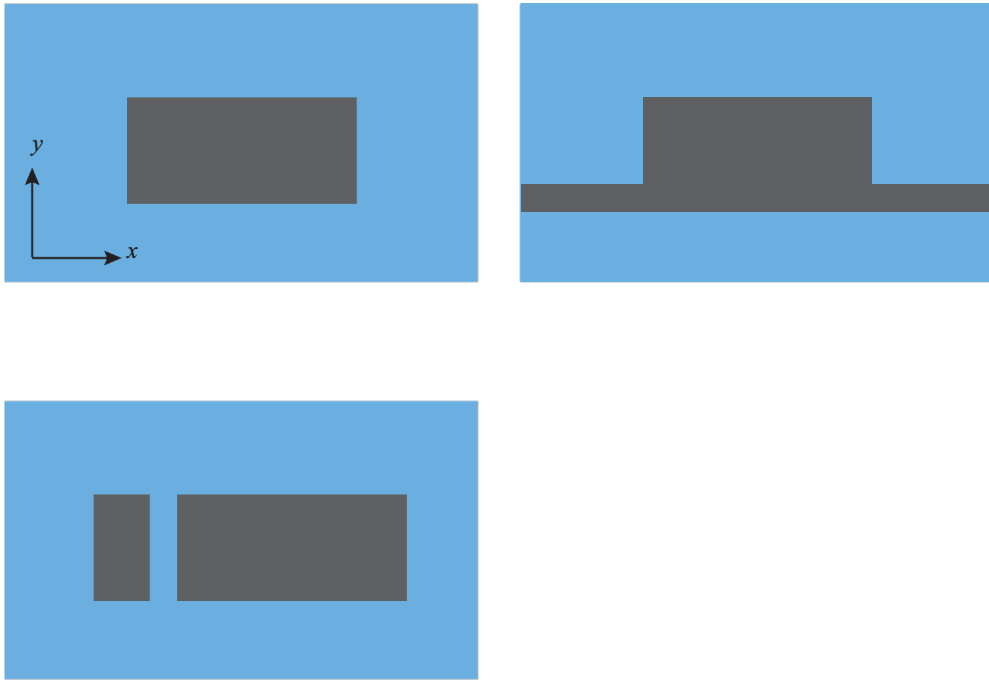
- Dispersion engineering: shaping of GVD profile in a targeted wavelength range
- For integrated waveguides: chose cladding, core height and core width to shape GVD
- How to obtain GVD?
 - Mode solvers (Lumerical, Comsol) calculate mode profiles, n_{eff} and associated wavevector β
 - Wavelength can be swept to obtain the dispersion of a given mode
 - Geometrical parameters can be swept to map the GVD for different structures

Rational for integration - *dispersion*

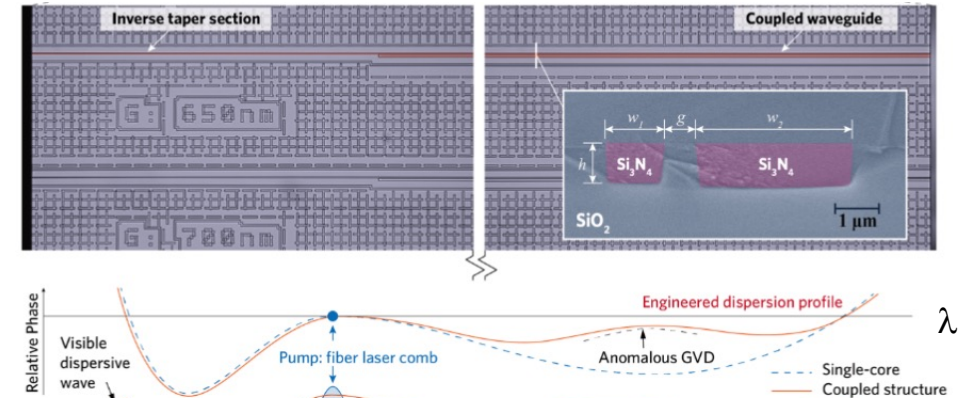


- Large core-cladding index contrast
 - Appearance of **2 ZDWs**
 - **Strong anomalous dispersion** window
- Small changes result in large ZDW(s)
 - Lithographic control on dispersion
- Such sensitivity is also a drawback!
 - Precise dispersion engineering is constrained by nanofabrication tolerances and maturity

Advanced dispersion engineering in integrated waveguides (1)

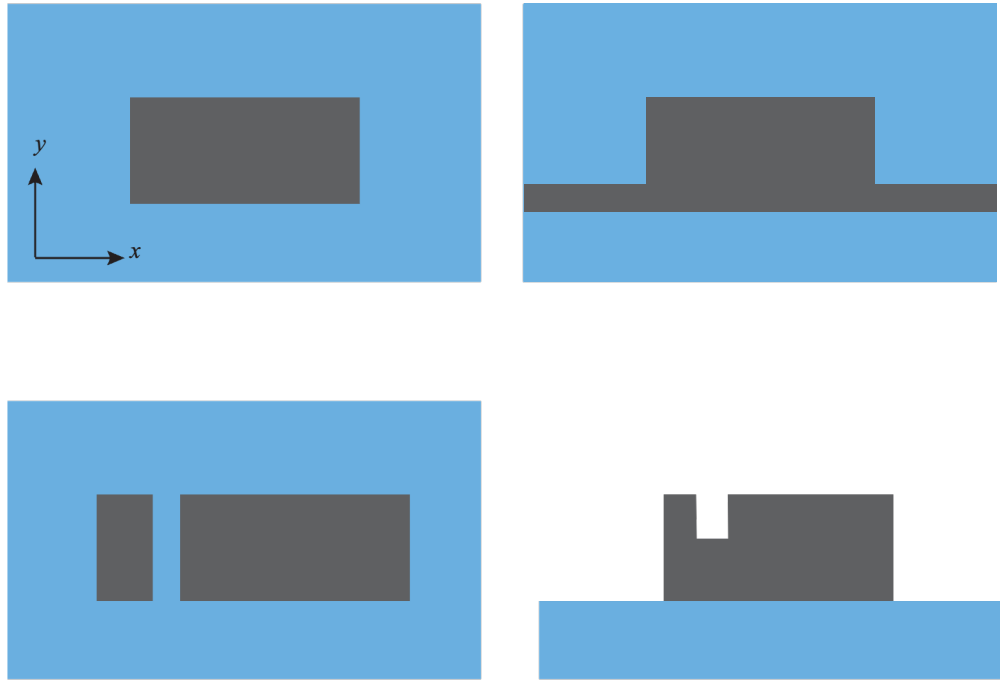


- Coupled waveguides involving supermodes
 - Additional lateral waveguide has strong effect at long λ

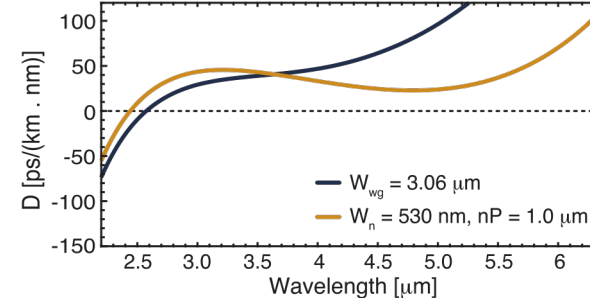
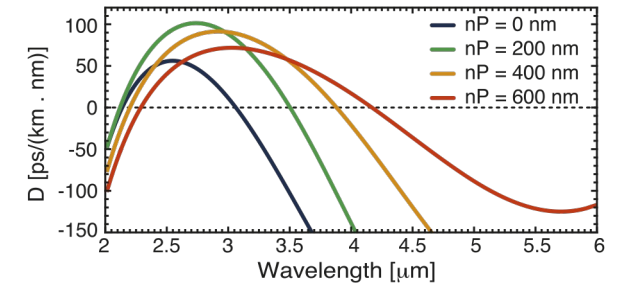
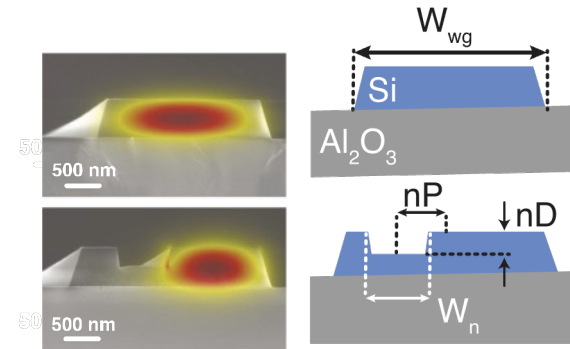


Hairun Guo, et al. "Nanophotonic supercontinuum-based mid-infrared dual-comb spectroscopy," **Optica** 7, 1181-1188 (2020)

Advanced dispersion engineering in integrated waveguides (2)



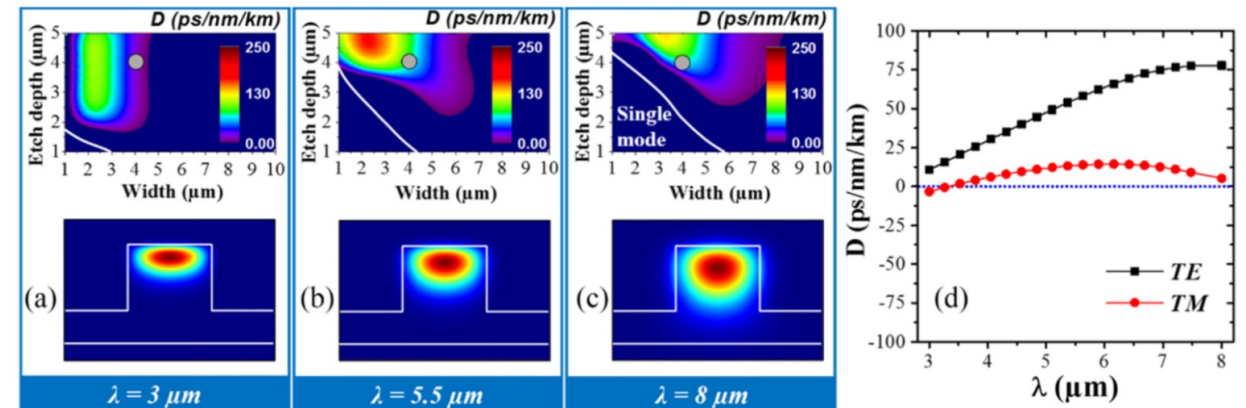
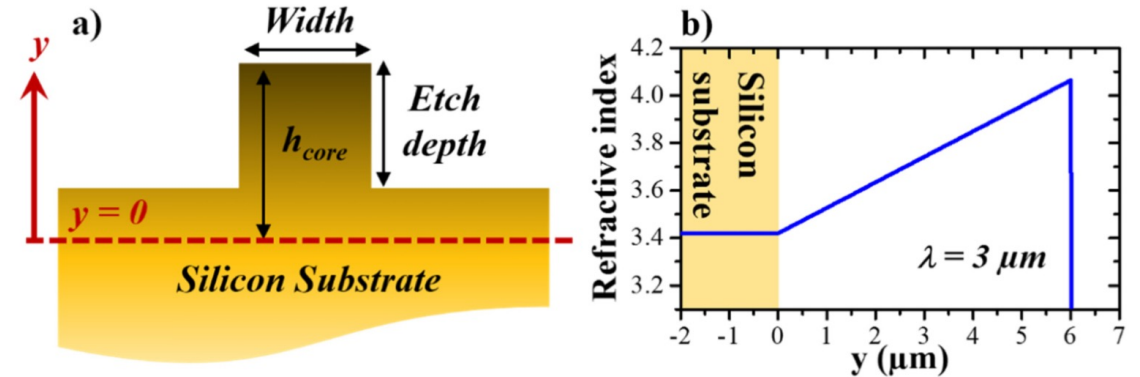
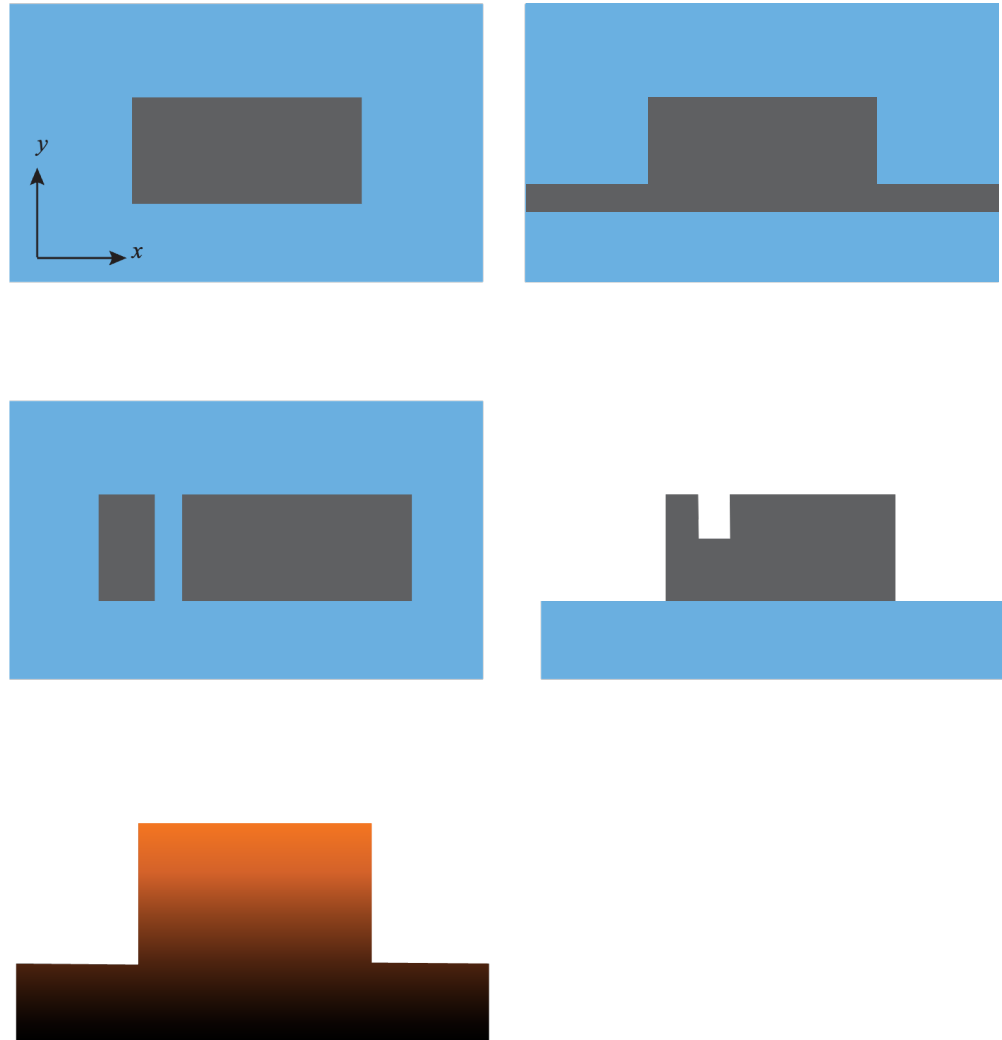
■ Notch waveguide



Nima Nader, et al. "Versatile silicon-waveguide supercontinuum for coherent mid-infrared spectroscopy," **APL Photonics** 3, 036102 (2018)

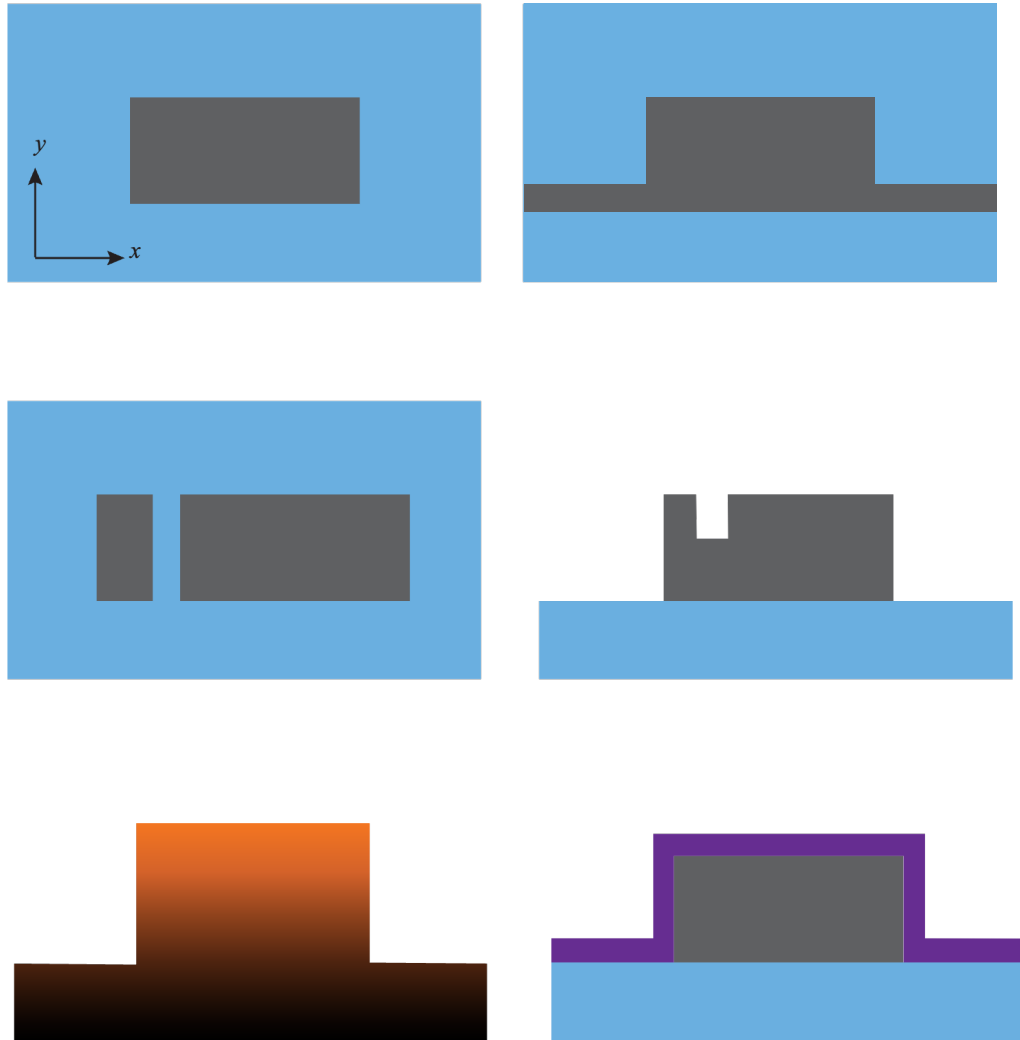
Advanced dispersion engineering in integrated waveguides (3)

- Graded index waveguides (SiGe)
 - Fine tune refractive index, control mode confinement

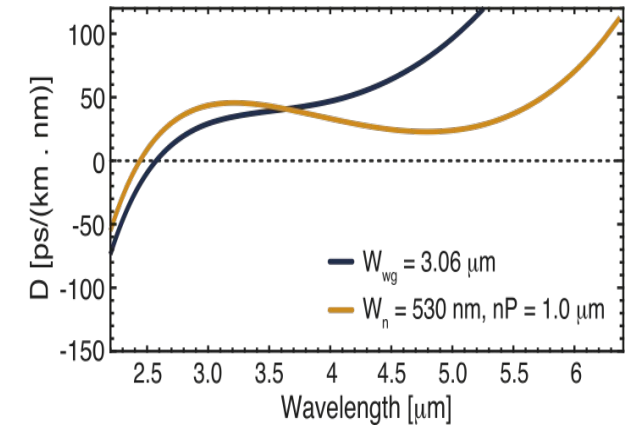
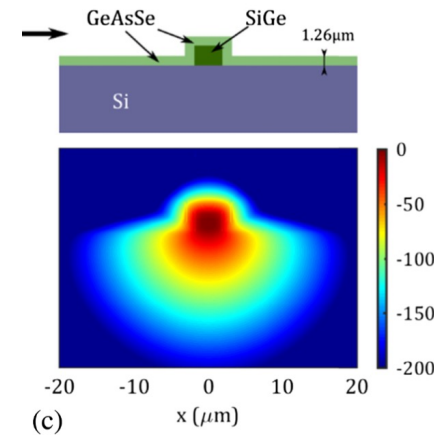


J. M. Ramirez, et al. "Ge-rich graded-index Si Ge waveguides with broadband tight mode confinement and flat anomalous dispersion for nonlinear mid-infrared photonics," **Opt. Express** 25, 6561-6567 (2017)

Advanced dispersion engineering in integrated waveguides (4)

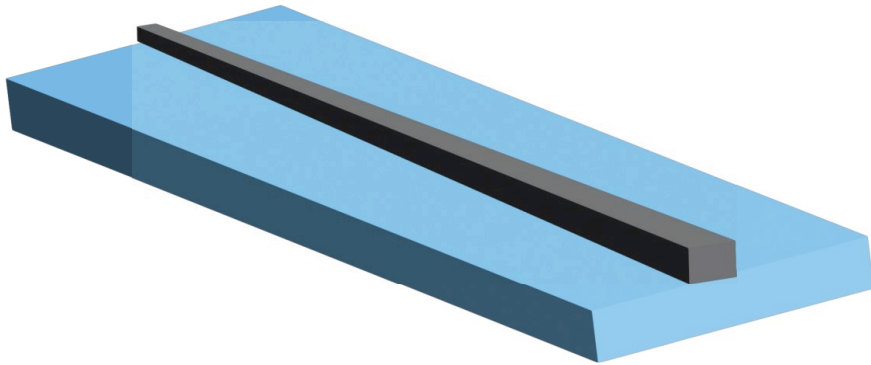


- Changing top 'cladding' of the waveguide
 - Can be a post process dispersion control
 - Tune the dispersion from anomalous to normal

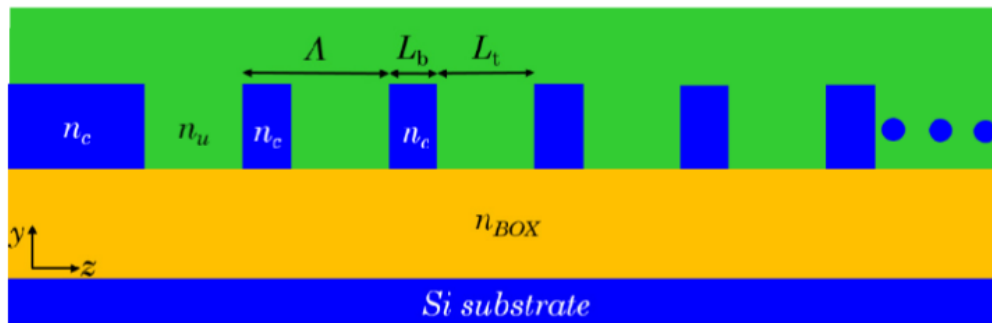


Milan Sinobad, et al. "Dispersion trimming for mid-infrared supercontinuum generation in a hybrid chalcogenide/silicon-germanium waveguide," **J. Opt. Soc. Am. B** 36, A98-A104 (2019)

Advanced dispersion engineering in integrated waveguides (5)



- Can easily vary the geometry along the waveguide length
 - 3D dispersion engineering
 - Continuously varying β_2
 - Cascaded section which can be combined with ML (J. Wei et al, PRA 14, 054045 2020)
 - Sub wavelength structures



D. Benedikovic, et al. Optics Express 25 (16) (2017)

In this lecture

Quick basic of integrated optics and intro to nonlinear optics in waveguides

- Theory of optical waveguide
- Important parameters for nonlinear optics
- Platforms of interest

Integrated nonlinear optics in waveguides – illustration with supercontinuum generation

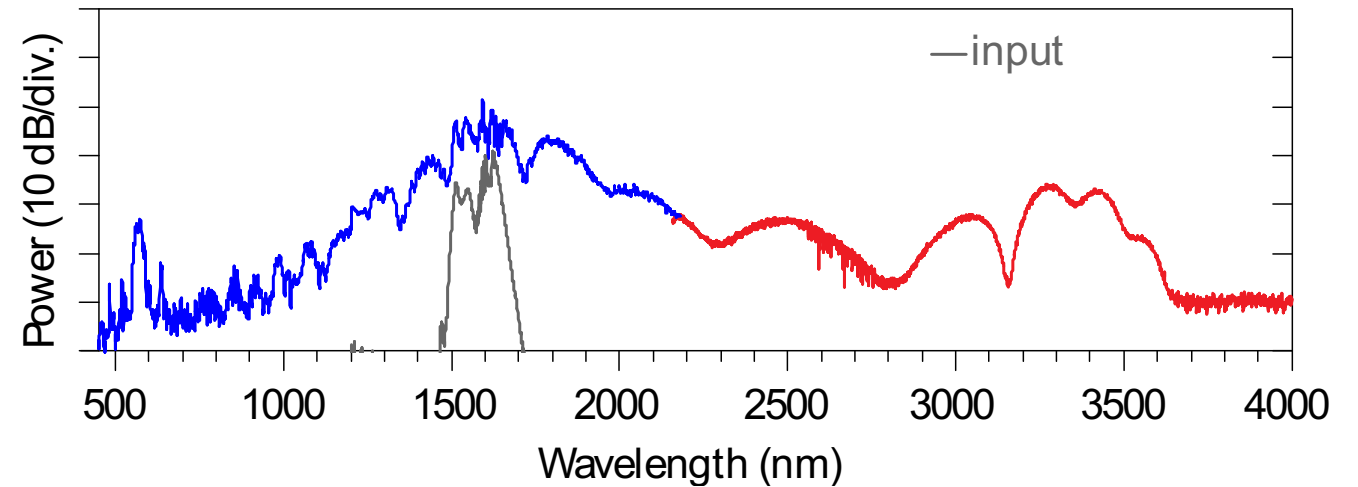
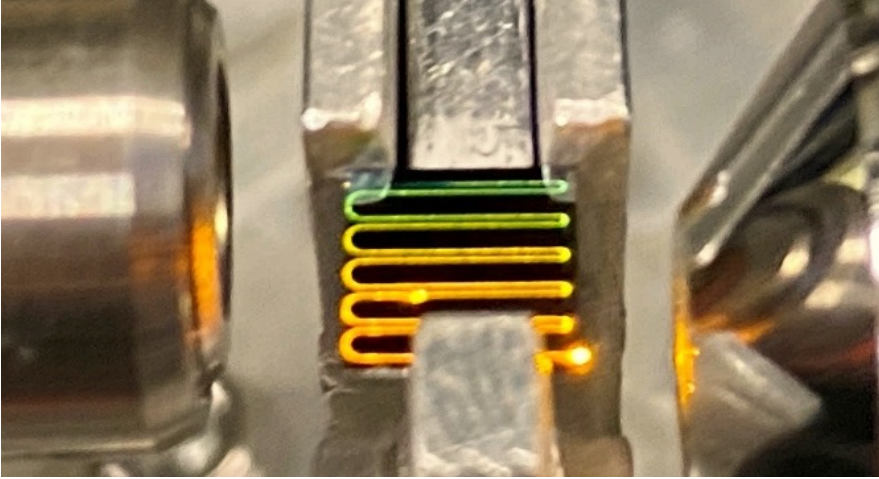
- Pulse propagation physics
- Design rules
- Demonstrations

Integrated nonlinear optics in microring resonators

- Basics of microresonator
- Nonlinear behavior
- Illustration of Kerr comb generation and dissipative solitons

Supercontinuum generation

Propagation of laser light in a nonlinear waveguide can result in extreme spectral broadening



Integrated photonics offer a reduction of footprint, cost and complexity

Enhanced light-matter interaction in diverse materials also resulted in **new design rules and opportunities** for supercontinuum generation

Supercontinuum generation

VOLUME 24, NUMBER 11

PHYSICAL REVIEW LETTERS

16 MARCH 1970

OBSERVATION OF SELF-PHASE MODULATION AND SMALL-SCALE FILAMENTS IN CRYSTALS AND GLASSES

R. R. Alfano* and S. L. Shapiro

Bayside Research Center of General Telephone & Electronics Laboratories Incorporated,
Bayside, New York 11360

(Received 10 December 1969)

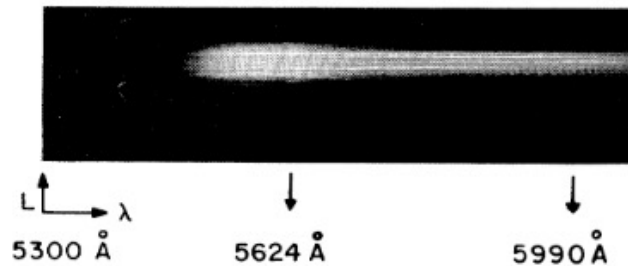
EMISSION IN THE REGION 4000 TO 7000 Å VIA FOUR-PHOTON COUPLING IN GLASS

R. R. Alfano and S. L. Shapiro

Bayside Research Center of General Telephone & Electronics Laboratories Incorporated,
Bayside, New York 11360

(Received 9 January 1970)

Bulk



Applied Physics Letters, Vol. 28, No. 4, 15 February 1976

New nanosecond continuum for excited-state spectroscopy

Chinlon Lin and R. H. Stolen

Bell Telephone Laboratories, Holmdel, New Jersey 07733

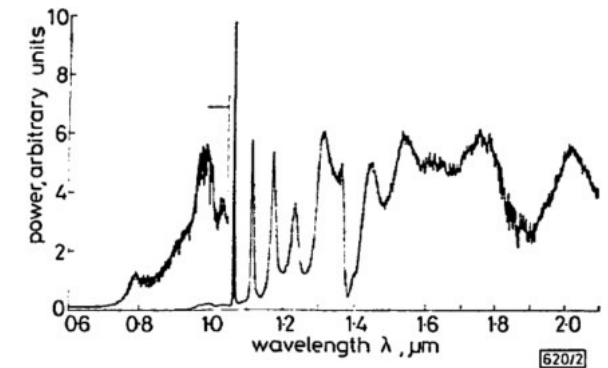
(Received 20 October 1975; in final form 1 December 1975)

ELECTRONICS LETTERS 7th December 1978 Vol. 14 No. 25

Wideband near-i.r. continuum (0.7-2.1 μm) generated in low-loss optical fibres

Chinlon Lin, V. Nguyen, W. French • Published 7 December 1978 • Physics • Electronics Letters

Fiber



Supercontinuum generation – quest for the broad

12 November 2007 / Vol. 15, No. 23 / OPTICS EXPRESS 15242

Supercontinuum generation in silicon photonic wires

I-Wei Hsieh,¹ Xiaogang Chen,¹ Xiaoping Liu,¹ Jerry I. Dadap,¹ Nicolae C. Panoiu,²
Cheng-Yun Chou,¹ Fengnian Xia,³ William M. Green,³ Yuri A. Vlasov,³ and
Richard M. Osgood, Jr.¹

¹Microelectronics Sciences Laboratories, Columbia University, New York, NY, 10027, USA

²Department of Electronic and Electrical Engineering, University College London, Torrington Place, London, UK

³IBM T. J. Watson Research Center, Yorktown Heights, NY 10598, USA

ih2109@columbia.edu

Integrated

15 September 2008 / Vol. 16, No. 19 / OPTICS EXPRESS 14938

Supercontinuum generation in dispersion engineered highly nonlinear ($\gamma = 10$ /W/m) As₂S₃ chalcogenide planar waveguide

Michael R.E. Lamont,¹ Barry Luther-Davies,² Duk-Yong Choi,² Steve Madden,²
and Benjamin J. Eggleton¹

¹Centre for Ultrahigh-bandwidth Devices for Optical Systems (CUDOS), School of Physics,
University of Sydney, NSW 2006, Australia

egg@physics.usyd.edu.au

²Centre for Ultrahigh-bandwidth Devices for Optical Systems (CUDOS), Laser Physics Centre,
The Australian National University, Canberra, ACT 0200, Australia

bld111@rsphysse.anu.edu.au

Supercontinuum generation

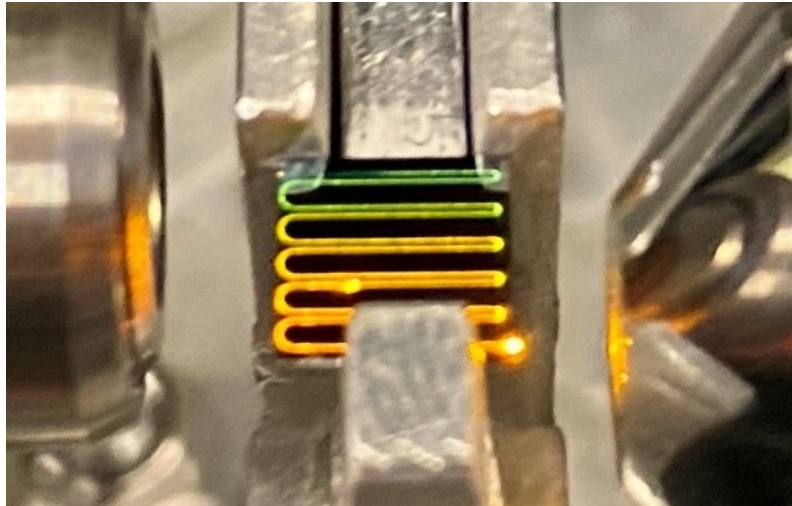
Propagation of laser light in a nonlinear waveguide can result in extreme spectral broadening



Frequency conversion process with wide optical spectrum

High coherence

'Simple' source



- Extremely useful solution to generate new optical frequencies.
 - Sometimes only viable solution to reach 'unconventional spectral windows'
- Spatial coherence and can have temporal coherence
 - Despite broad bandwidth : frequency comb
- Powerful technique to bridge distant spectral regions
 - Does not require resonant structure,
 - Does not require additional seed nor temporal synchronization

What about the reach, efficiency and control of supercontinuum for a given pump source ?

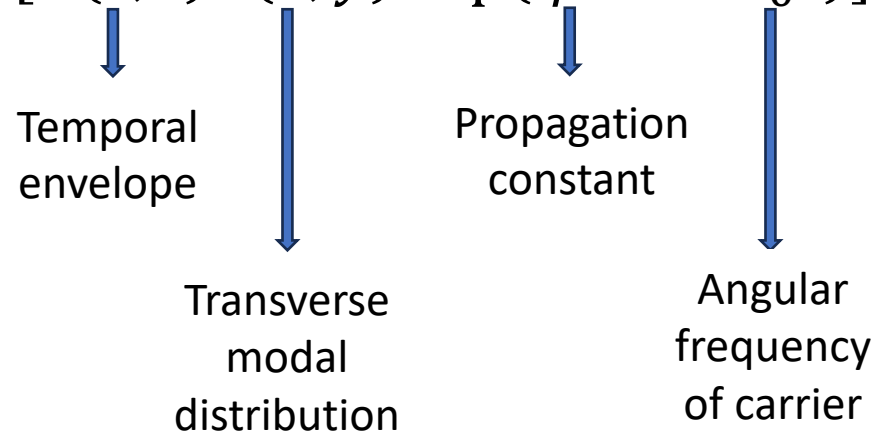
Nonlinear polarization

$$\mathbf{P}(\mathbf{r}, t) = \mathbf{P}_L(\mathbf{r}, t) + \mathbf{P}_{NL}(\mathbf{r}, t)$$

$$\nabla^2 \mathbf{E} - \frac{n_0^2}{c_0^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{1}{\epsilon_0 c_0^2} \frac{\partial^2 \mathbf{P}_{NL}}{\partial t^2}$$

- Consider envelope-based approach and propagation in the z direction

$$\mathbf{E}(\mathbf{r}, t) = \text{Re}[A(z, t) \mathbf{F}(x, y) \exp(i\beta z - i\omega_0 t)]$$



Propagation physics

- Considering only cubic nonlinearity ($\mathbf{P}_{NL} \approx \varepsilon_0 \chi^{(3)} : \mathbf{E}\mathbf{E}\mathbf{E}$) of the core and neglecting term oscillating at $3\omega_0$:

$$\mathbf{P}_{NL}(\mathbf{r}, t) \approx \frac{3}{4} \varepsilon_0 \chi^{(3)} |A(z, t)|^2 A(z, t) |\mathbf{F}(x, y)|^2 \mathbf{F}(x, y)$$


$$\frac{\partial A(z, t)}{\partial z} - \sum_{k \geq 2} \frac{i^{k+1}}{k!} \beta_k \left(\frac{\partial^k A(z, t)}{\partial t^k} \right) + \frac{\alpha}{2} A(z, t) = i\gamma |A(z, t)|^2 A(z, t)$$

With:

$$\gamma = \frac{\omega_0 n_2}{c_0 A_{eff}} \quad n_2 = \frac{3\chi^{(3)}}{4\varepsilon_0 c_0 n_0^2} \quad A_{eff} = \frac{\left(\iint_{-\infty}^{\infty} |\mathbf{F}(x, y)|^2 dx dy \right)^2}{\iint_{\sigma_3} |\mathbf{F}(x, y)|^4 dx dy}$$

GNLSE

Raman contribution

$$R(t) = (1 - f_r)\delta(t) + f_r h_r(t)$$

Frequency dependence of γ :

$$\gamma(\omega) \approx \gamma(\omega_0) + \gamma_1(\omega - \omega_0) + \dots$$

$$\gamma_1 = \left. \frac{d\gamma}{d\omega} \right|_{\omega=\omega_0} \approx \frac{1}{\omega_0}$$

$$\frac{\partial A(z, t)}{\partial z} - \underbrace{\sum_{k \geq 2} \frac{i^{k+1}}{k!} \beta_k \left(\frac{\partial^k A(z, t)}{\partial t^k} \right)}_{\text{Dispersion}} + \underbrace{\frac{\alpha}{2} A(z, t)}_{\text{Loss}} = \underbrace{i\gamma \left(1 + i\gamma_1 \frac{\partial}{\partial t} \right)}_{\text{Self-steepening}} \left(A(z, t) \underbrace{\int_{-\infty}^{\infty} R(t') |A(z, t - t')|^2 dt'}_{\text{SPM, FWM, Raman}} \right)$$

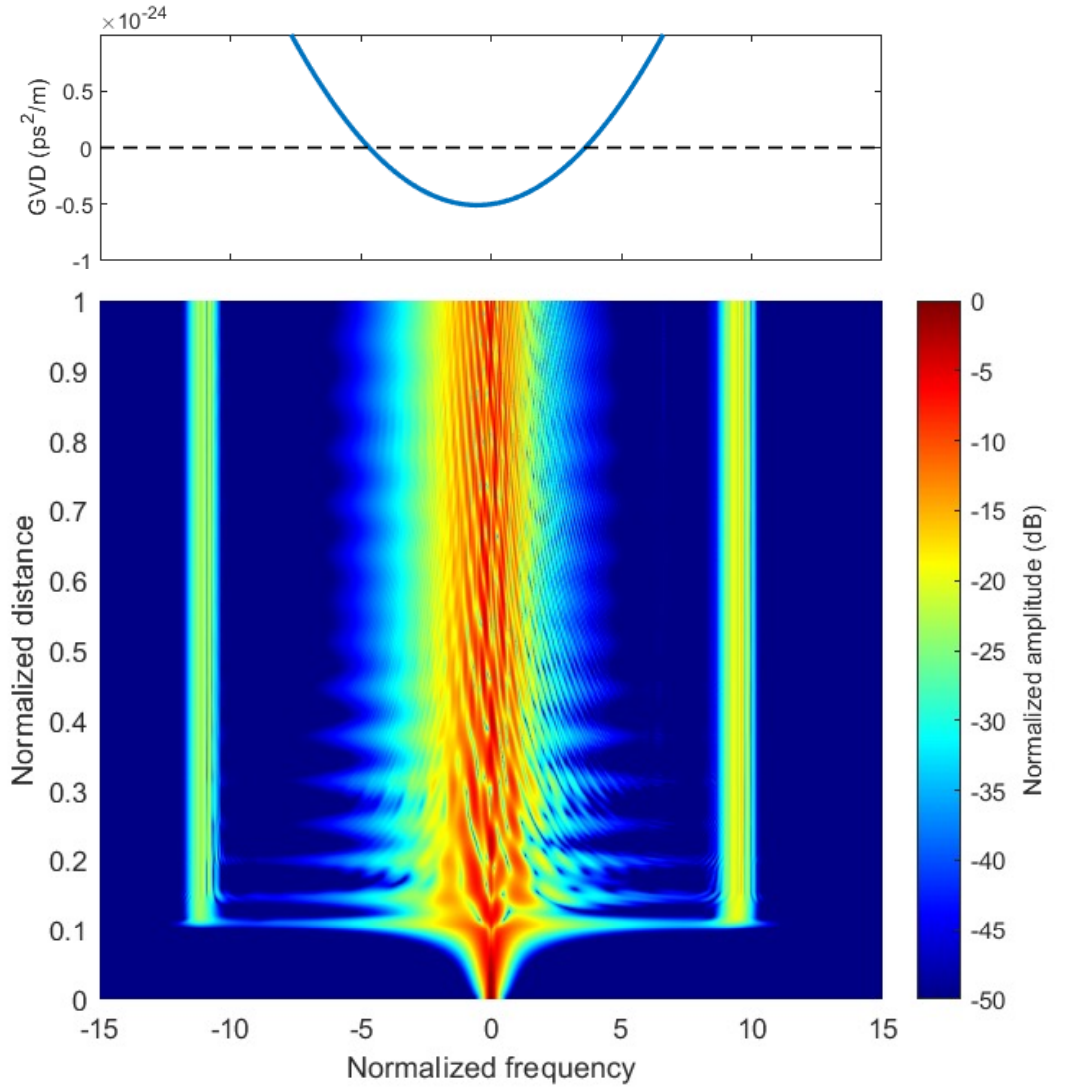
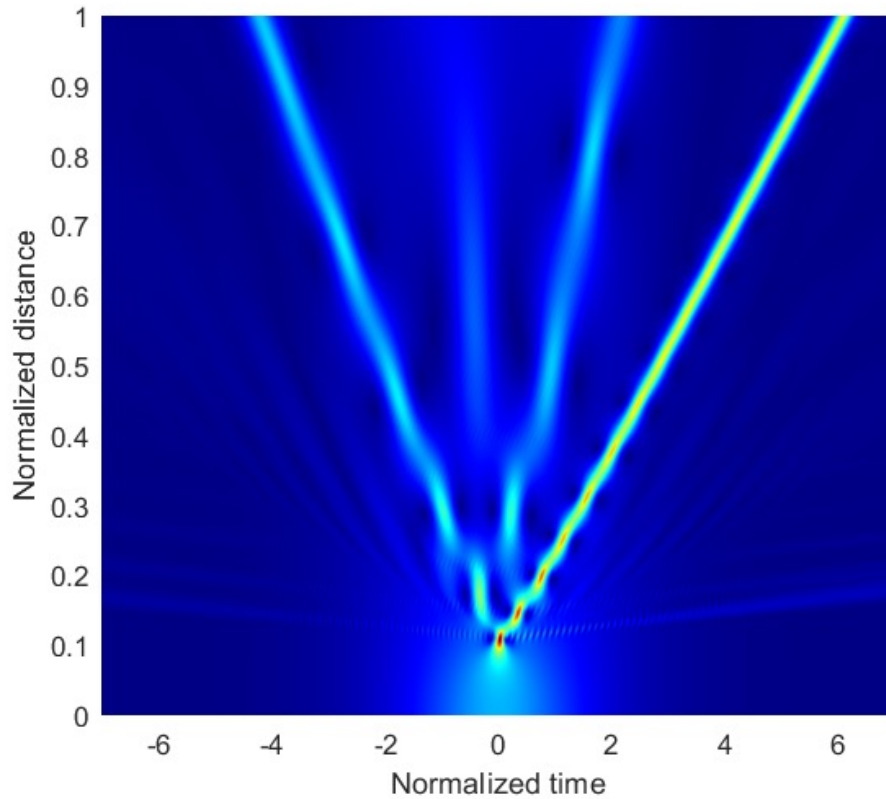
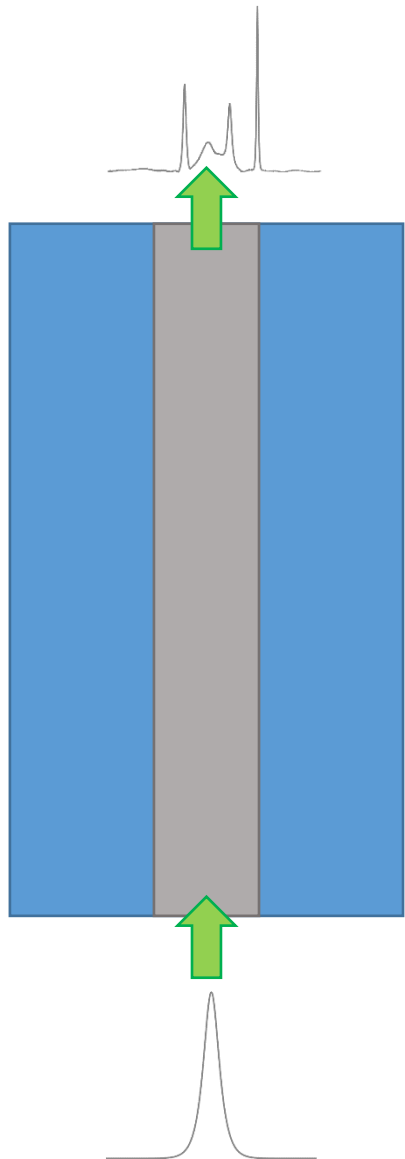
Propagation in anomalous dispersion

- A large majority of work on SCG relies on pumping in the **anomalous dispersion**
 - Leverages opposite effects of SPM and GVD that govern the generation of solitons and their dynamics
- Soliton regime requires that $L_D \geq L_{NL}$. Pulses are characterized by soliton number N

$$N = \sqrt{\frac{L_D}{L_{NL}}} = \sqrt{\frac{\gamma P_0 T_0^2}{|\beta_2|}}$$

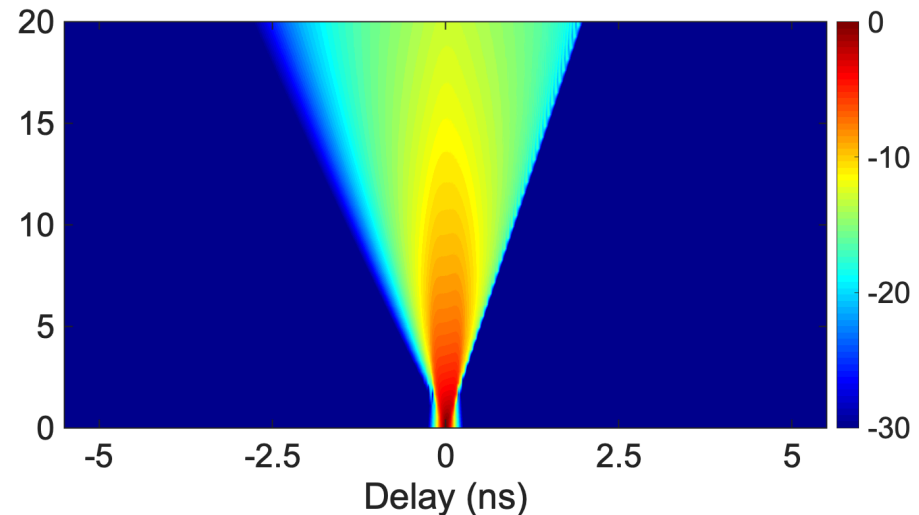
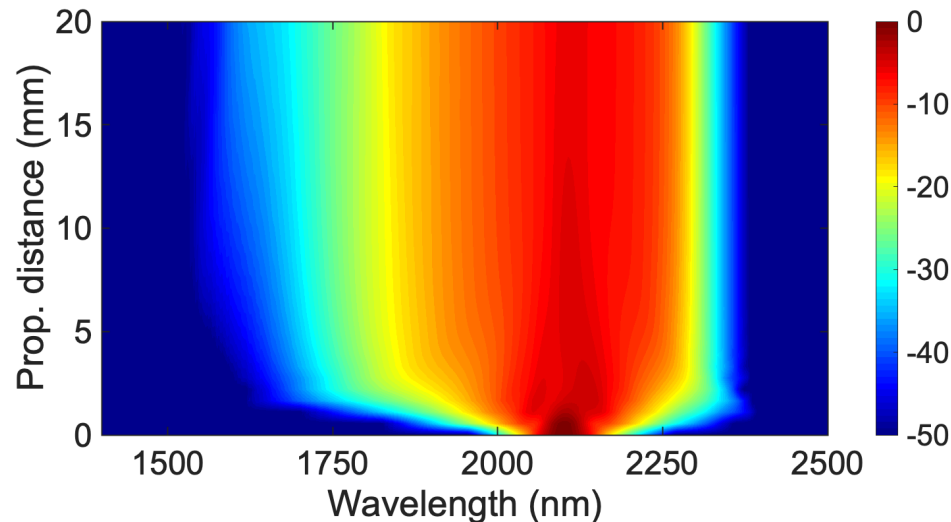
- For $N > 1$, higher order solitons experience temporal compression points.
 - Higher order soliton are easily perturbed, can lead to soliton fission (N fundamental solitons split in time)
- The fission length is given by : $L_{\text{fiss}} \approx \frac{L_D}{N} = \frac{T_0}{\sqrt{\gamma P_0 |\beta_2|}}$

Propagation in anomalous dispersion



Propagation in normal dispersion

- Main mechanism is SPM: broadening is typically narrower
 - dispersion-induced temporal stretching is no longer compensated by counteracting Kerr effect
- Optical wave breaking (OWB) generates the extreme wavelength components
 - Shifted components overlap with non-shifted components at the edges of the pulse
 - Leads to non-phase-matched generate FWM if peak of the pulse is still sufficiently high



Important concept - coherence

- A SC can have pulse-to-pulse stability: high degree of temporal coherence, frequency comb

Coherence in anomalous regime

- Can have modulation instability (MI)

$$\Omega_{\text{MI}} = \sqrt{\frac{2\gamma P_0}{|\beta_2|}}$$

- Want soliton dynamics to dominate MI
 - $N < 22$, J.C. Travers J. Opt. 12 113001 (2010)
 - $N < 16$, J.M. Dudley, et al. Rev. Mod. Phys., 78:1135, 2006
 - Typically means that pulses have to be < 100 fs

Coherence in normal regime

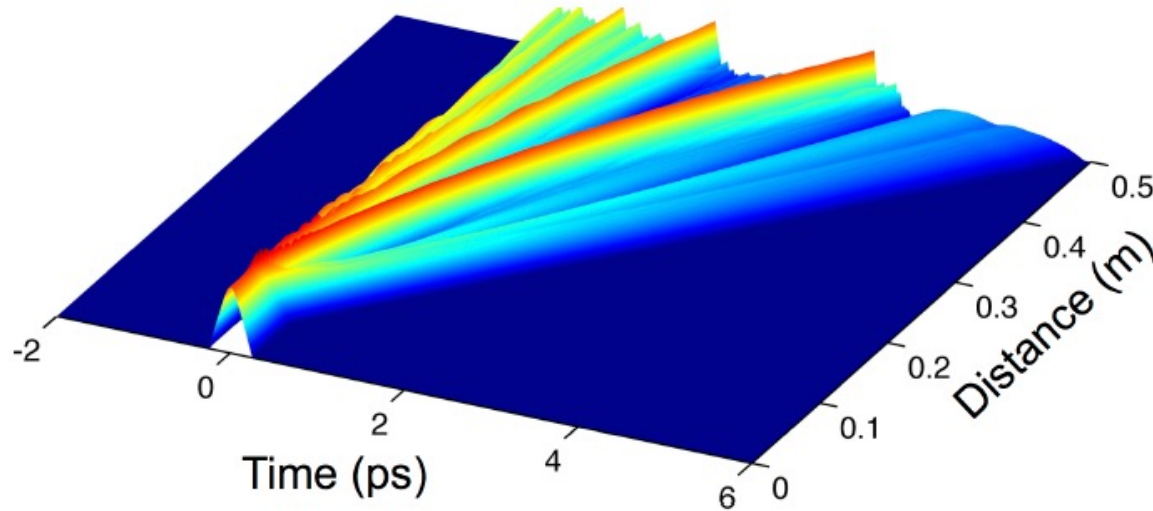
- SPM/OWB: less susceptible to noise amplification
- Coherence can still be degraded
 - From non-phase-matched parametric amplification of noise from nonlinear coupling of SRS and FWM
- Occurs for $N > 600$ A. Heidt, et al. JOSA B 4(4):764 (2017)
 - Can have coherent supercontinuum with ps pulses

Often estimated through numerical simulations:

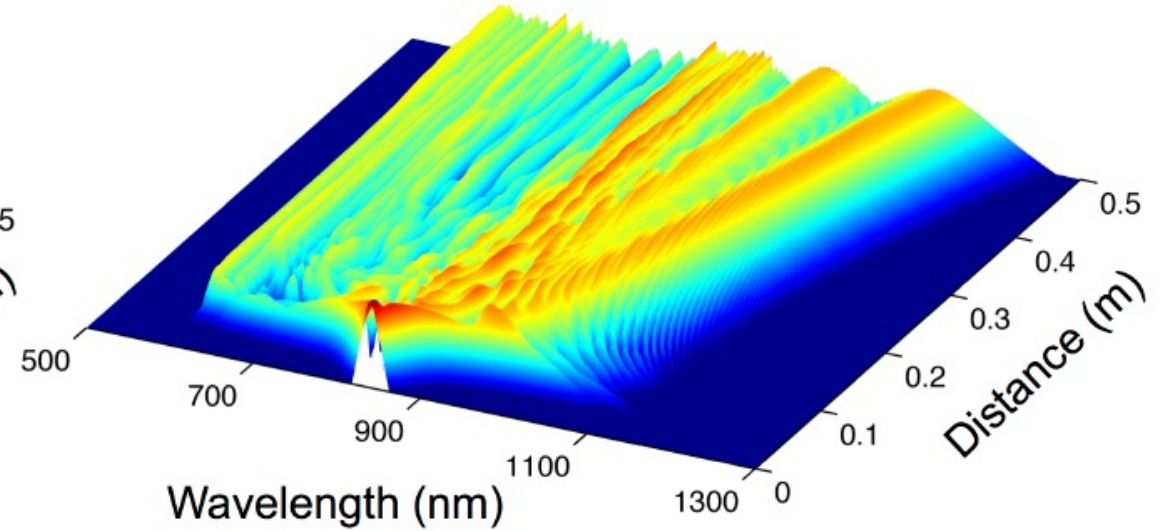
$$|g_{12}^{(1)}(\lambda)| = \frac{\langle E_1^*(\lambda) E_2(\lambda) \rangle}{\sqrt{\langle |E_1(\lambda)|^2 \rangle \langle |E_2(\lambda)|^2 \rangle}}$$

Supercontinuum generation regimes

Time evolution



Spectral evolution



Long pulses (ns)

Short pulses (fs)

Anomalous dispersion

Modulation instability
Soliton dynamics

Soliton dynamics
Dispersive waves

Normal dispersion

Raman scattering
Four-wave mixing

Self-phase modulation
Four-wave mixing

Summary of important parameters

Parameter

Expression

Nonlinear coefficient

$$\gamma = \frac{\omega_0 n_2}{c_0 A_{\text{eff}}}$$

Dispersion length

$$L_D = \frac{T_0^2}{|\beta_2|}$$

Nonlinear length

$$L_{\text{NL}} = \frac{1}{\gamma P_0}$$

Effective length

$$L_{\text{eff}} = \frac{1 - \exp(-\alpha L)}{\alpha}$$

Soliton order

$$N = \sqrt{\frac{L_D}{L_{\text{NL}}}} = \sqrt{\frac{\gamma P_0 T_0^2}{|\beta_2|}}$$

Parameter

Expression

MI angular frequency

$$\Omega_{\text{MI}} = \sqrt{\frac{2\gamma P_0}{|\beta_2|}}$$

Fission length

$$L_{\text{fiss}} \approx \frac{L_D}{N} = \frac{T_0}{\sqrt{\gamma P_0 |\beta_2|}}$$

OWB length (Gaussian pulse)

$$L_{\text{OWB}} \approx 1.1 \frac{L_D}{N}$$

Coherence limit
(anomalous SCG)

$$N \approx 22$$

Coherence limit
(normal SCG)

$$N \approx 600$$

Examples of SCG in integrated optics

Material	width x height	Pump (nm)	γ (W ⁻¹ m ⁻¹)	α (dB/cm)	coupling loss
SiO ₂ HNLF [1]	Core radius ~ 1.7 μ m	1690	0.0194	N.A.	0.7 dB/ splice
SiO ₂ PCF [2]	Core radius 1.13 μ m	1064	0.023	$< 10^{-5}$	3 dB/facet
SOI [3]	1.6 x 0.39 μ m ²	2300	38	0.2	12 dB/facet
SOS [4]	3.45 x 0.66 μ m ²	3060	N.A. (~ 9 from other source)	5 – 7	~ 7 dB/facet
Si ₃ N ₄ [5]	0.69 x 0.95 μ m ²	1030	1	0.7	11 dB/facet
Si ₃ N ₄ [6]	1.175 x 2.29 μ m ²	2090	0.37	0.2	5.5 dB/facet
Si _{0.6} Ge _{0.4} [7]	5 x 2.7 μ m ²	4000	0.72	0.3	4.2 dB/facet
AlGaAs [8]	0.5 x 0.3 μ m ²	1555	630	2	12 dB/facet
AlN [9]	0.42 x 0.5 μ m ²	780	9.8	6	4 dB/facet
TFLN [10]	0.8 x 0.8 μ m ²	1506	N.A. (~ 0.4 from other source)	3	8.5 dB/facet

⇒ Compact structures (mm length)
 ⇒ High nonlinearity
 ⇒ Wide range of wavelength

⇒ High propagation losses
 ⇒ High coupling losses

[1] Takashi Hori et al. **Optics Express**, 12(2):317–324, 2004

[2] Alexander M. Heidt et al. **Optics Express**, 19(4):3775–3787, 2011

[3] Bart Kuyken et al. **Nature Communications**, 6(1):6310, 2015

[4] Nima Nader et al. **APL Photonics**, 3(3):036102, 2018

[5] Adrea R. Johnson et al. **Optics Letters**, 40(21):5117, 2015

[6] Davide Grassani et al. **Nature Communications**, 10(1):1553, 2019

[7] Milan Sinobad et al **Optica**, 5(4):360, 2018.

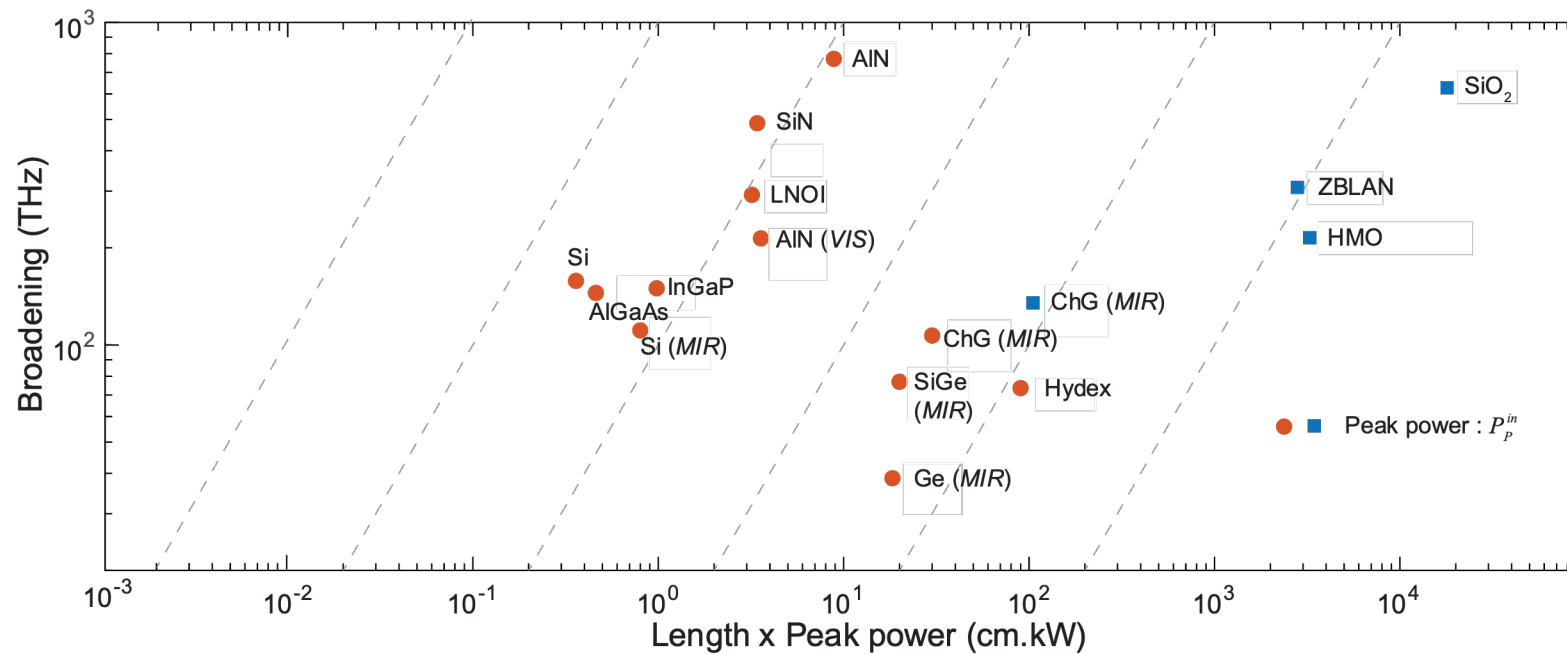
[8] Bart Kuyken et al. **Optics Letters**, 45(3):603–606, 2020

[9] Xianwen Liu et al. **Nature Communications**, 10(11):2971, 2019

[10] Mengjie Yu et al. **Optics Letters**, 44(5):1222, 2019

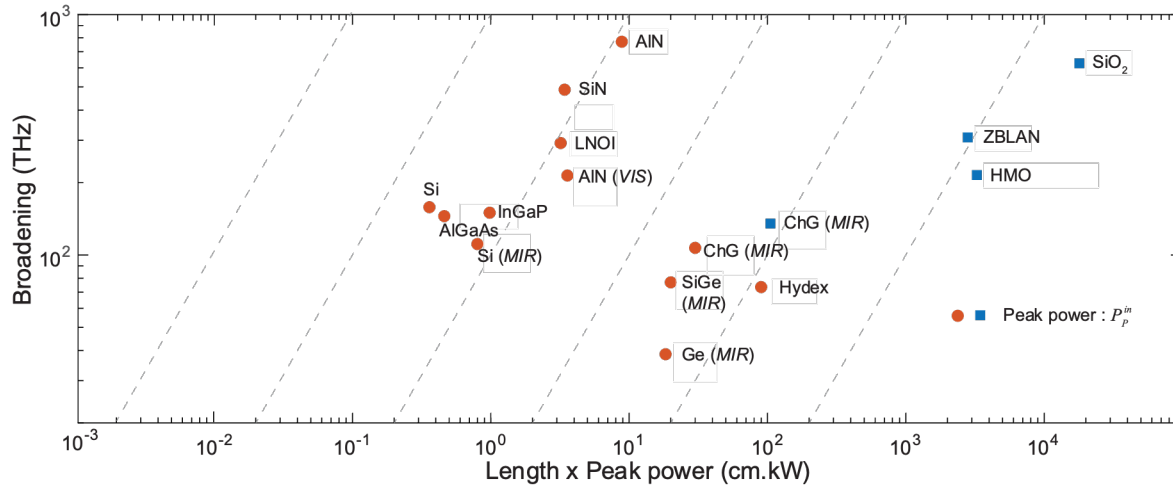
SCG in integrated optics

- Proposed figure of merit for supercontinuum generation: $FOM_{SCG} = \frac{\Delta\nu}{LP_0}$



SiO₂ PCF: J. M. Stone et al., *Opt. Express*, vol. 16, pp. 2670, Feb 2008.
HMO PCF: A. N. Ghosh et al., *JOSA. B*, vol. 35, pp. 2311, Sep 2018.
ZBLAN fiber: C. Xia et al., *Opt. Lett.*, vol. 31, pp. 2553, Sep 2006.
ChG nanowire: D. D. Hudson et al., *Optica*, vol. 4, pp. 1163, Oct 2017.
Hydrex: D. Duchesne et al., *Opt. Express*, vol. 18, pp. 923, Jan 2010.
SiN: H. Guo et al., *Nature Photonics*, vol. 12, pp. 330, 2018
Si: N. Singh et al., *Light: Science & Applications*, vol. 7, pp. 17131, 2018
Si (MIR): N. Nader et al., *Optica*, vol. 6, pp. 1269, Oct 2019.
SiGe: M. Montesinos-Ballester et al., *ACS Photonics*, vol. 7, pp. 3423, 2020.
Ge: A. Della Torre et al., *APL Photonics*, vol. 6, p. 016102, 2021.
ChG: Y. Yu et al., *Opt. Lett.*, vol. 41, pp. 958, Mar 2016.
AlN(VIS): X. Liu et al., *Nature Communications*, vol. 10, no. 2971, 2019.
AlN: J. Lu et al., *Opt. Lett.*, vol. 45, pp. 4499, Aug 2020.
AlGaAs: B. Kuyken et al., *Opt. Lett.*, vol. 45, pp. 603, Feb 2020.
InGaP: U. D. Dave et al., *Opt. Express*, vol. 23, pp. 4650, Feb. 2015
LNOI: E. Obrzud et al., *APL Photonics*, vol. 6, no. 12, p. 121303, 2021.

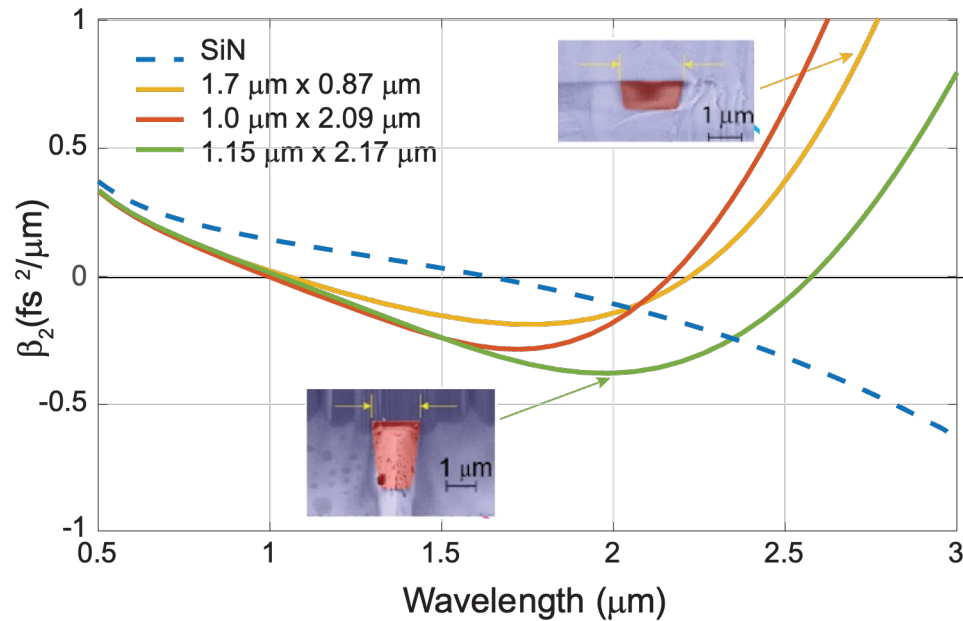
SCG in integrated waveguides – first conclusions



- Obtained record broadenings in waveguide/fiber are similar, especially in near-IR
- FOM of integrated platforms can be several orders of magnitude higher than fiber platforms
 - Even when using the input pump power requirement

- ➡ Comes from significantly reduced *length x pump power*: advantage of chip-scale platforms
- ➡ For ultrabroad SCG, propagation loss are not a drawback
- ➡ Many platforms have similar FOM !

Engineering aspect – strong anomalous pumping



- Recall specifics of waveguide dispersion:

- Relatively large values of anomalous $|\beta_2|$

$$L_D = \frac{T_0^2}{|\beta_2|} \quad L_{\text{fiss}} \approx \frac{T_0}{\sqrt{\gamma P_0 |\beta_2|}} \quad \Omega_{\text{MI}} = \sqrt{\frac{2\gamma P_0}{|\beta_2|}}$$

- Large anomalous dispersion window between 2 ZDWs

➡ Ultrabroad band anomalous SCG in integrated photonics is often bounded by dispersive waves located in normal dispersion regime

Dispersive wave generation

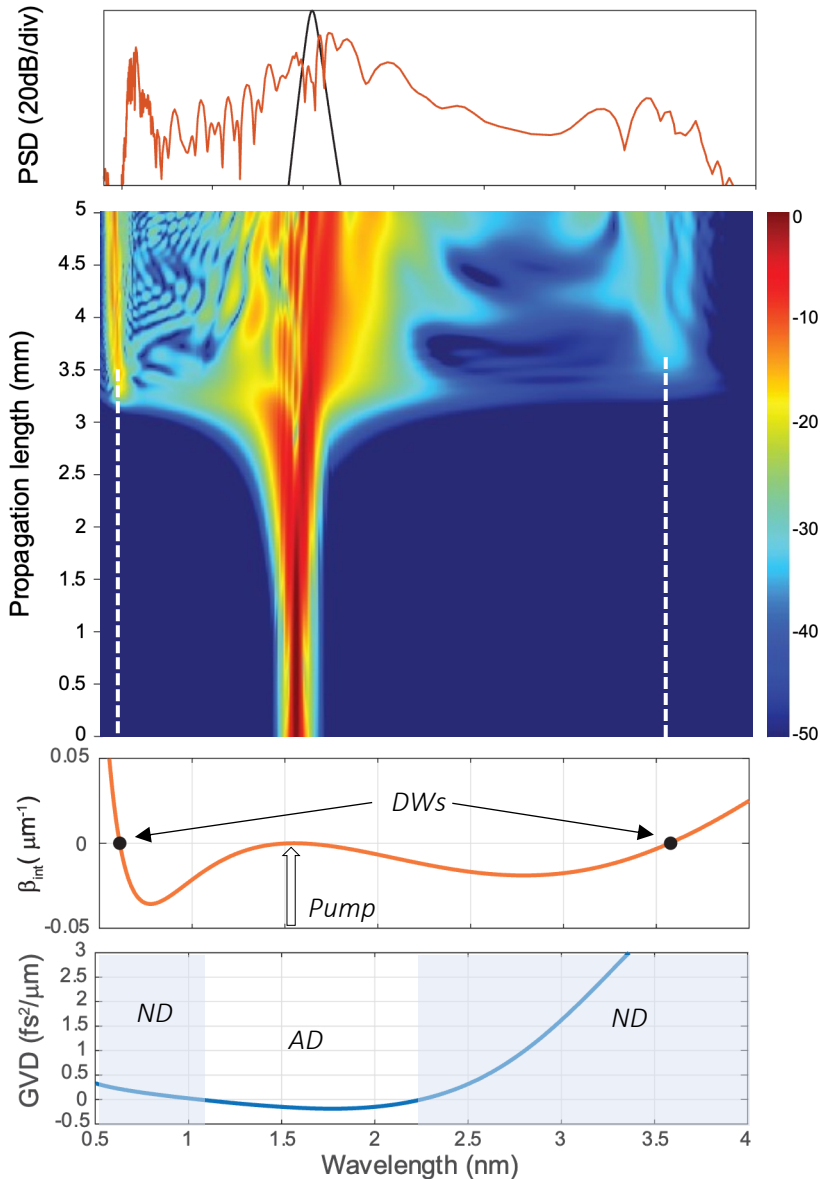
- Femtosecond pulse in anomalous dispersion
 - Spectrum is broadened through high order soliton dynamics
 - Soliton can be perturbed by HOD, shedding energy to a phase matched DW located in normal dispersion regime
- Phase matching condition: phase constant of soliton ω_s equals that of the linear wave at ω_D

$$\beta(\omega_s) - v_g^{-1}\omega_s + \frac{\gamma P}{2} = \beta(\omega_D) - v_g^{-1}\omega_D$$

- For negligible soliton nonlinear phase compared to $\beta(\omega)$ we are looking for frequencies ω where:

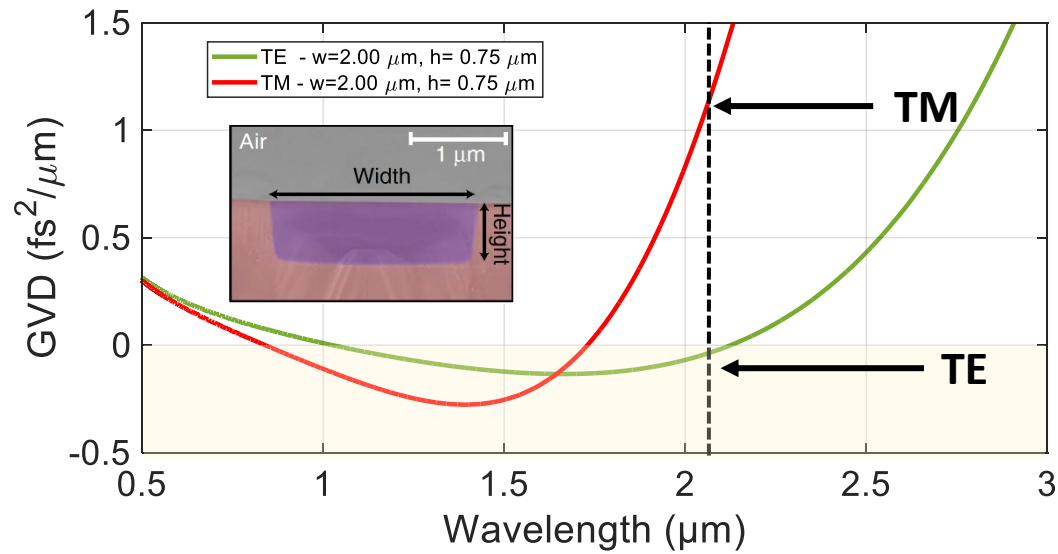
$$\underbrace{\beta(\omega) - \beta(\omega_s) - v_g^{-1}(\omega - \omega_s)}_{\Delta\beta(\omega)} \approx 0$$
$$\Delta\beta(\omega) \approx \sum_{m \geq 2} \frac{(\omega - \omega_s)^2}{m!} \frac{d^m \beta(\omega)}{d\omega^m} \triangleq \beta_{int}$$

DW engineering & designs rules in integrated waveguides



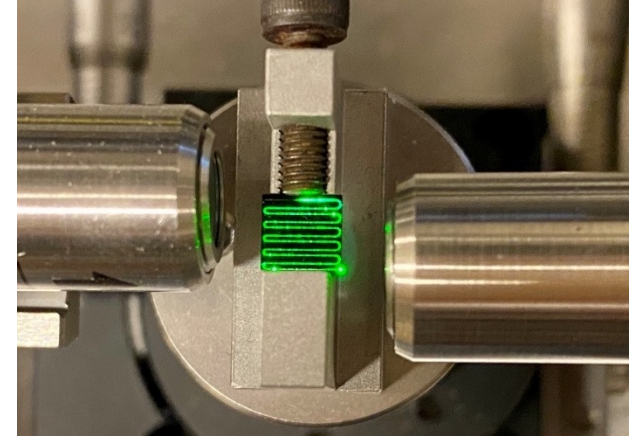
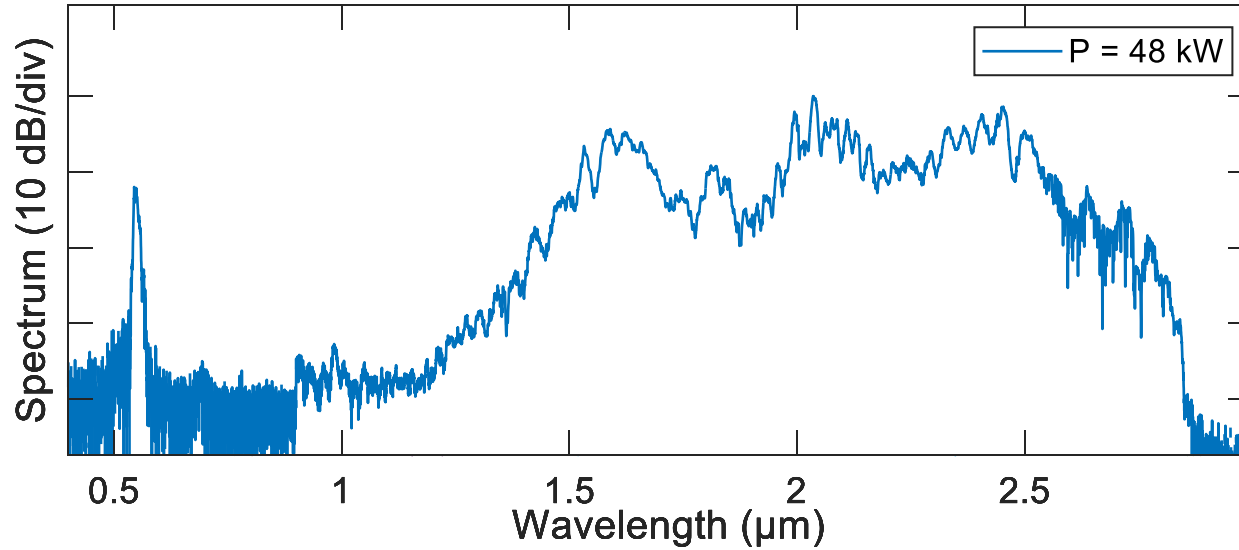
- Typical design rules for broadband SCG in integrated waveguide differ from that of fiber platforms
- In fiber: low $|\beta_2|$ at the pump λ + long propagation length
 - Broadening depends on the soliton number N
 - For finite P_0 and given T_0 , a low γ platform implies low value of $|\beta_2|$
 - All characteristics length become long ($L_{\text{fiss}}...$)
- In integrated waveguides: γ can be much higher
 - Can have higher values of $|\beta_2|$, allows pumping away from ZDW
 - Can have two well-separated DWs with high coherence
 - Reduces all main physical length scales : device lengths of few mm

Engineering aspect – anomalous and normal SCG

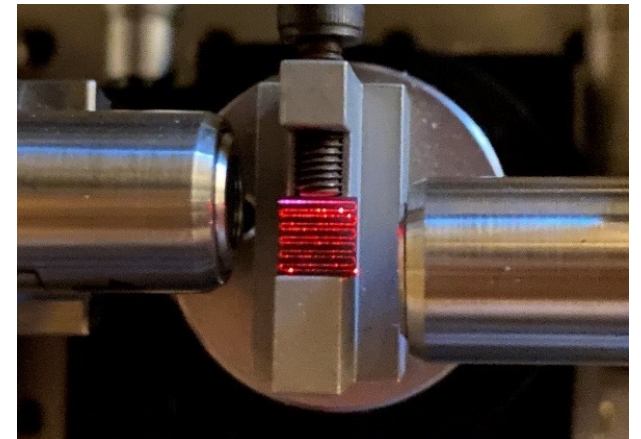
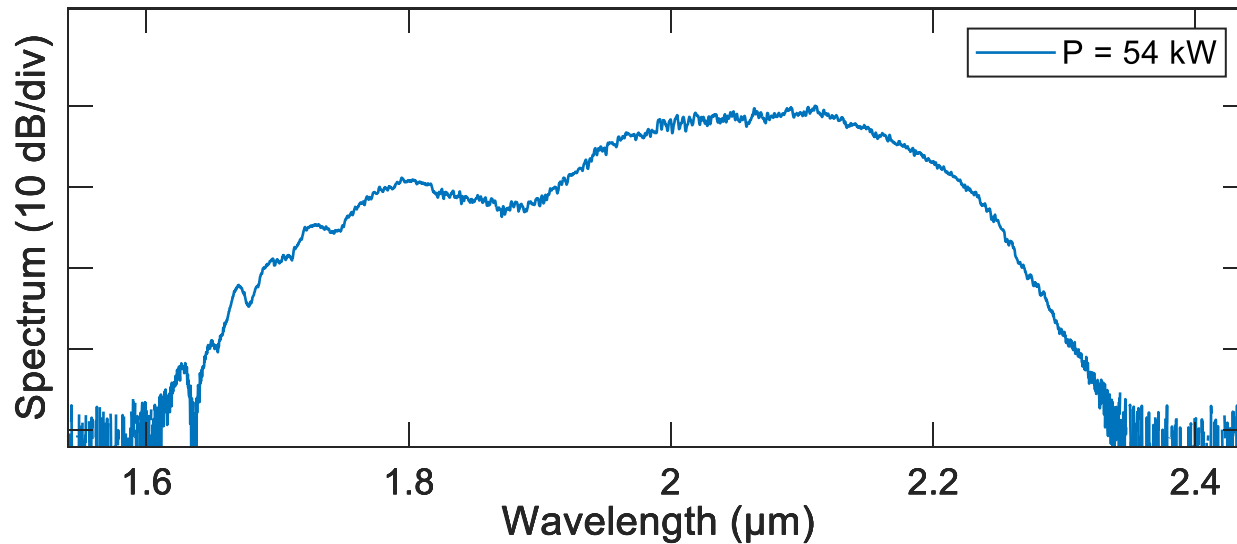


Engineering aspect – anomalous and normal SCG

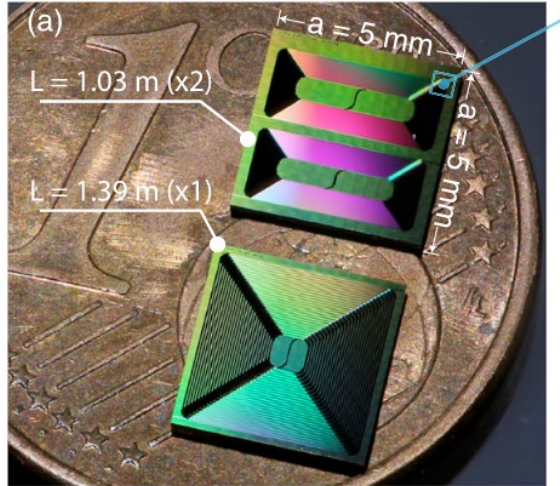
TE polarization



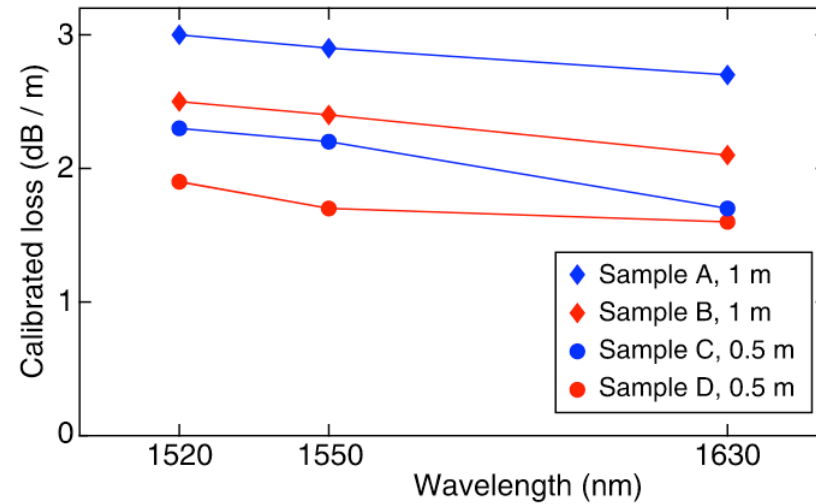
TM polarization



Trend: Pushing the performance of 'low' γ platform (SiN)

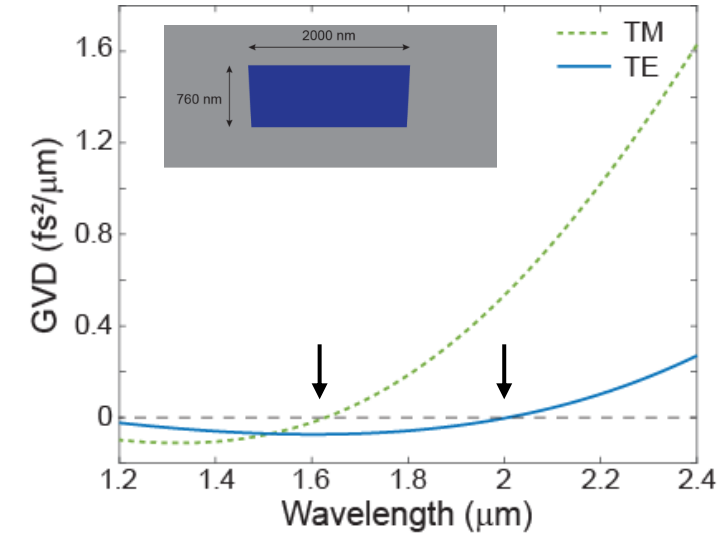


Meter-long waveguides



Ultra low losses

J Liu, et al. Nature Com 12 (2021)



Low dispersion engineering

- Other trend: use high γ waveguide that can also be dispersion engineered !
- GaP waveguide
 - $165\text{ W}^{-1}\text{m}^{-1}$ (35-fold higher than SiN), propagation losses 1 dB/cm
 - See N. Kuznetsov et al, arXiv:2404.08609

In this lecture

Quick basic of integrated optics and intro to nonlinear optics in waveguides

- Theory of optical waveguide
- Important parameters for nonlinear optics
- Platforms of interest

Integrated nonlinear optics in waveguides – illustration with supercontinuum generation

- Pulse propagation physics
- Design rules
- Demonstrations

Integrated nonlinear optics in microring resonators

- Basics of microresonator
- Nonlinear behavior
- Illustration of Kerr comb generation and dissipative solitons

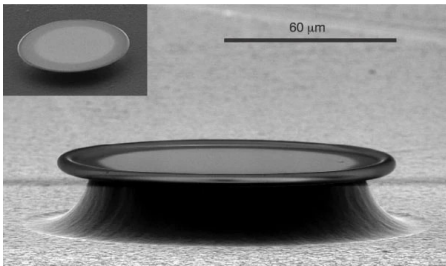
Optical microresonator

Confine light:

- Spatially, small mode volume - *large overlap and interaction strength of light with matter*
 - With low losses – *coherent enhancement of the interaction*

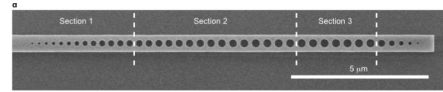
The concept of an optical microresonator can be implemented in different structures/geometries

WGM in cylindrical symmetric structure

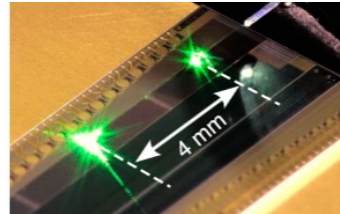


D. Armani, et al. Nature 421, p 925 (2003)

Integrated Fabry Perot resonator

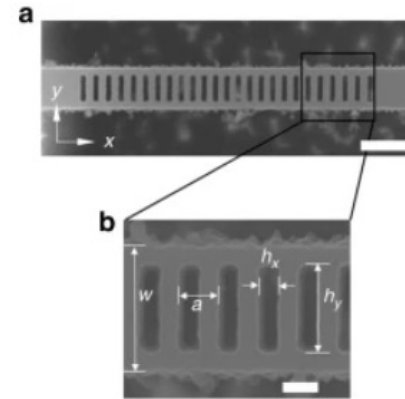


A. Nardi et al. arXiv:2304.12968v2



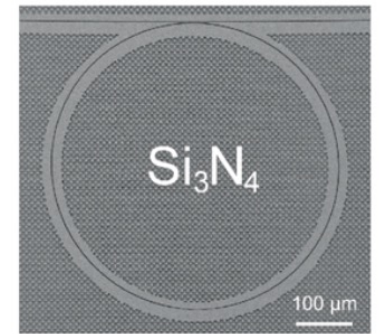
T. Wildi et al. arXiv:2206.10410v3

Photonic Crystals



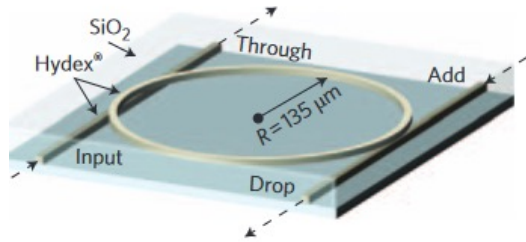
S. Kim et al. Nature Com 9, 2623 (2018)

Closed loop waveguide
Microring resonator



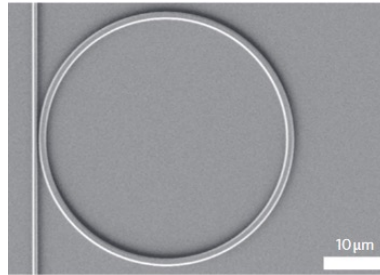
From Klab, EPFL

Microring resonators



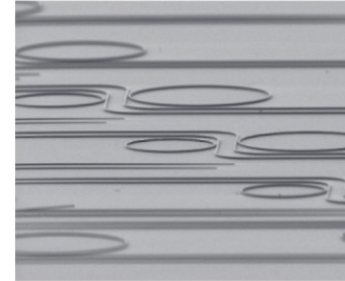
Hydex

Razzari et al. *Nature Photonics* 2009



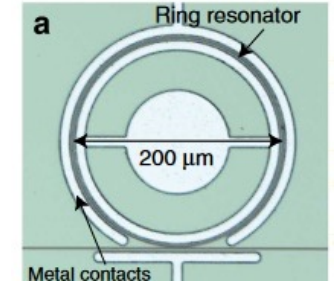
SiN

Levy et al. *Nature Photonics* 2010



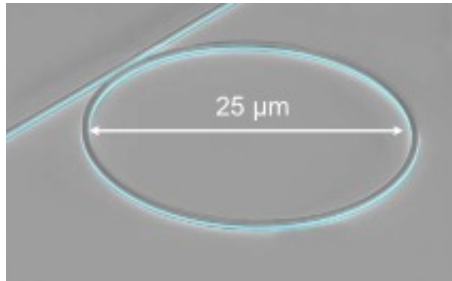
Diamond

Hausmann et al. *Nature Photonics* 2014



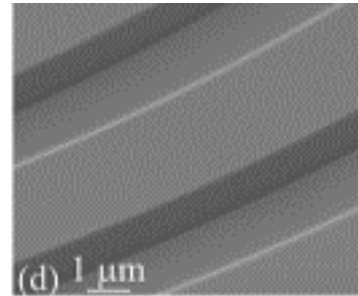
Si

Griffith et al. *Nature Com* 2015



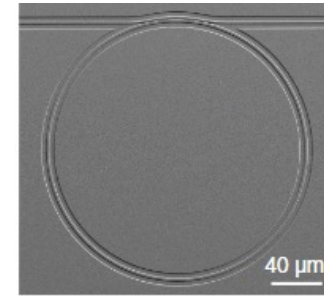
AlGaAs

Pu et al. *Optica* 2016



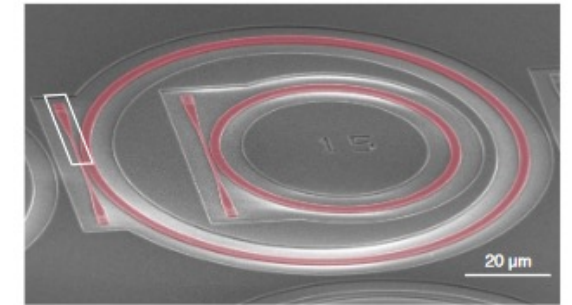
AlN

Liu et al. *ACS* 2018



LiNbO₃

He et al. *Optica* 2019



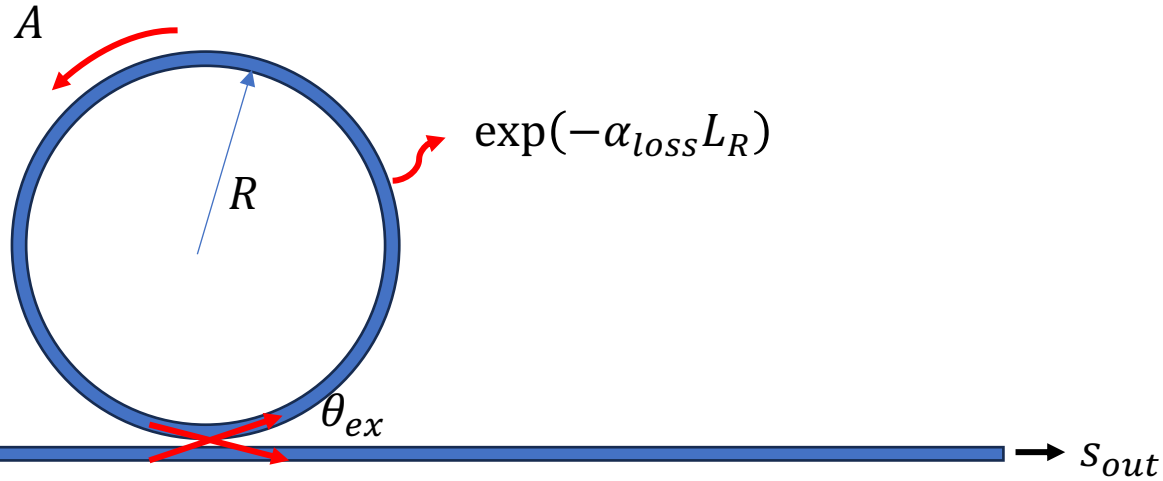
SiC

Guidry et al. *Optica* 2020

Linear coupling to a microring resonator

Resonator modes

$$\omega_m = m \frac{c}{n(\omega)R}$$



$|A|^2$ is the energy stored in the resonator

$|s_{in}|^2$ represents the photon flux of the pump and relates to the incident power.

Rate equation

$$\frac{dA}{dt} = -\left(\frac{\kappa}{2} + i\delta\omega\right) A(t) + \sqrt{\kappa_{ex}} s_{in}(t), \text{ with } \delta\omega \text{ detuning between resonance and input pump frequency}$$

Loss channels:

$$\kappa_0 \approx \alpha_{loss} L_R / T_R$$

$$\kappa_{ex} \approx \theta_{ex} / T_R$$

Total energy loss:

$$\kappa = \kappa_0 + \kappa_{ex}$$

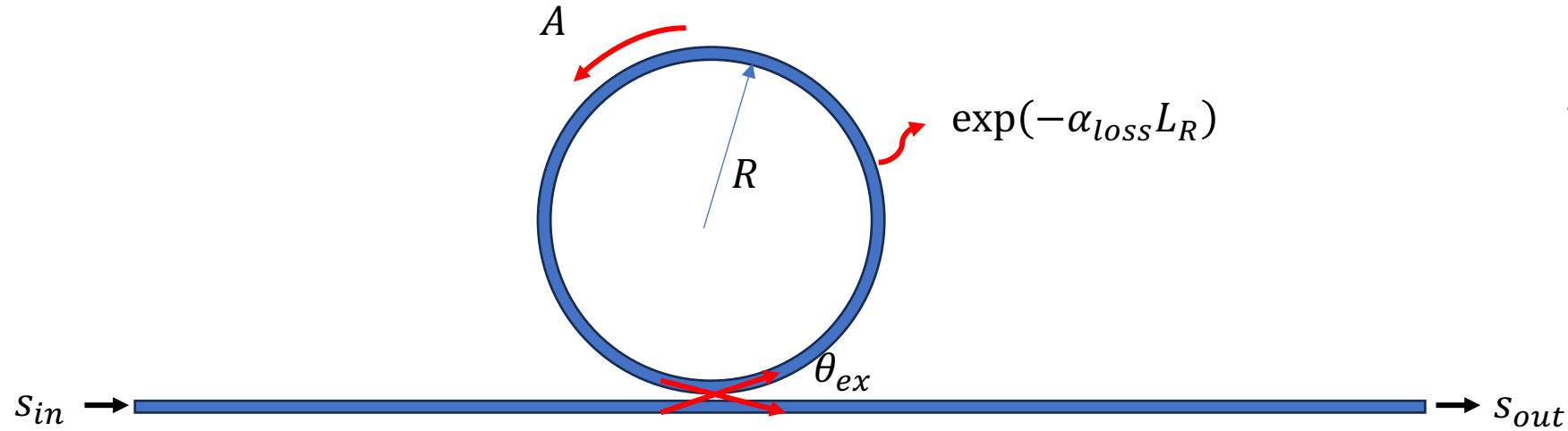
Coupling ratio

$$\eta = \kappa_{ex} / \kappa$$

Quality factor

$$Q = \omega_m / \kappa$$

Linear coupling to a microring resonator



Steady state transmission:

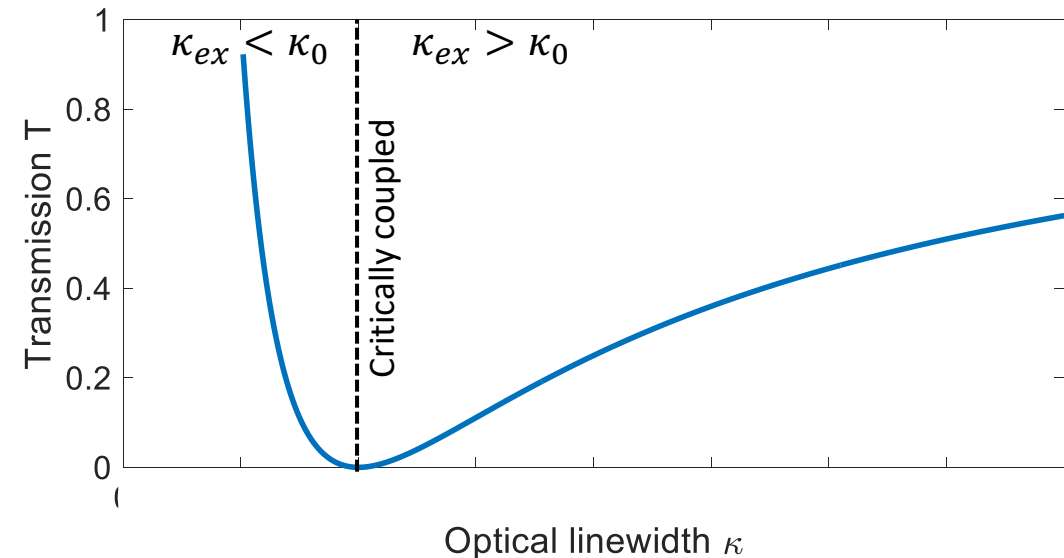
$$\left| \frac{S_{out}}{S_{in}} \right|^2 = 1 - \frac{\kappa_{ex} \kappa_0}{\left(\frac{\kappa}{2} \right)^2 + \delta \omega^2}$$

- Lorentzian dip with minimal transmission on resonance

- FWHM linewidth of $\kappa/2\pi$ (in frequency)
- Transmission depth $\Delta T = \left(\frac{\kappa_0 - \kappa_{ex}}{\kappa_0 + \kappa_{ex}} \right)^2$

- Coupling conditions $\eta = \kappa_{ex}/\kappa$

- Undercoupling: $\eta < \frac{1}{2}$ ($\kappa_{ex} < \kappa_0$)
- Critical coupling: $\eta = \frac{1}{2}$ ($\kappa_{ex} = \kappa_0$)
- Overcoupling: $\eta > \frac{1}{2}$ ($\kappa_{ex} > \kappa_0$)



Dispersion

- Frequencies of longitudinal modes are described by:

$$\omega_m = m \frac{c}{n(\omega)R}$$

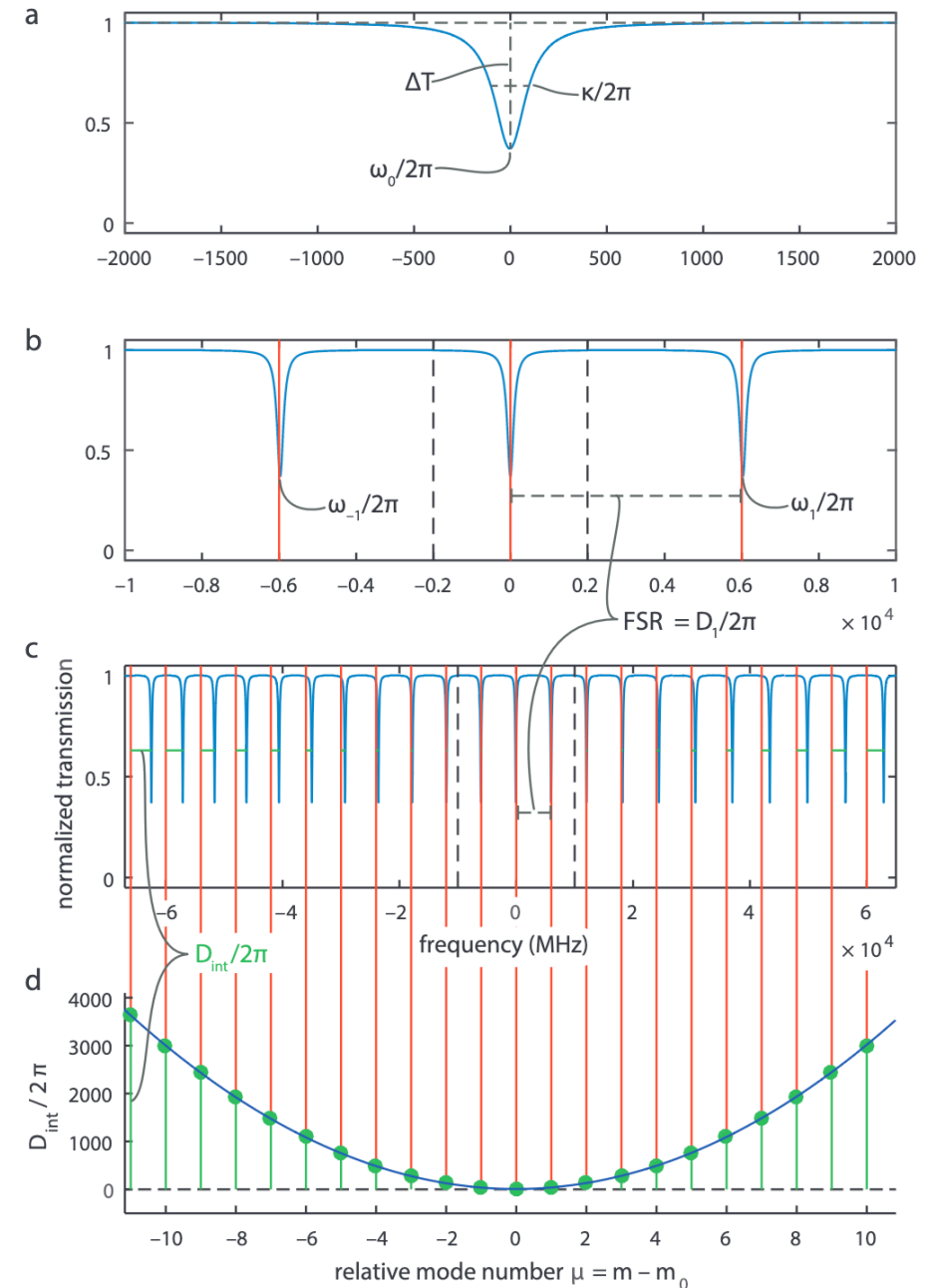
- Taylor expansion:

$$\omega_\mu \approx \omega_0 + \sum_{j \in \mathbb{N}} \frac{D_j}{j!} \mu^j$$

- Deviation from an equidistant grid is compounded into the integrated dispersive cavity mode shift:

$$D_{int}(\omega_\mu) = \omega_\mu - \omega_0 - D_1 \mu$$

$$D_{int}(\omega_\mu) \approx \frac{D_2}{2!} \mu^2 + \frac{D_3}{3!} \mu^3 + \dots$$



Nonlinear response

- Coupled-mode theory can be extended to describe many modes and nonlinear interactions

$$\frac{dA}{dt} = i \frac{D_2}{2} \frac{\partial^2 A}{\partial \phi^2} + ig|A|^2 A - \left(\frac{\kappa}{2} + i\delta\omega \right) A(t) + \sqrt{\kappa_{ex}} s_{in} \psi$$

$$g = \frac{\hbar \omega_0^2 c n_2}{n_0^2 V_{eff}}, \text{ per photon Kerr shift}$$

$$V_{eff} = A_{eff} L_R, \text{ effective mode volume}$$



$$\frac{d\psi}{dT} = -(1 + i\zeta_0)\psi + i|\psi|^2\psi - i\beta \frac{\partial^2 \psi}{\partial \Theta^2} + f$$

$$\psi = \sqrt{\frac{2g}{\kappa}} A$$

$$T = \frac{\kappa}{2} t$$

$$\Theta = \sqrt{\frac{\kappa}{2D_2}} \phi$$

$$f = \sqrt{\frac{2\eta g}{\kappa^2}} s_{in}$$

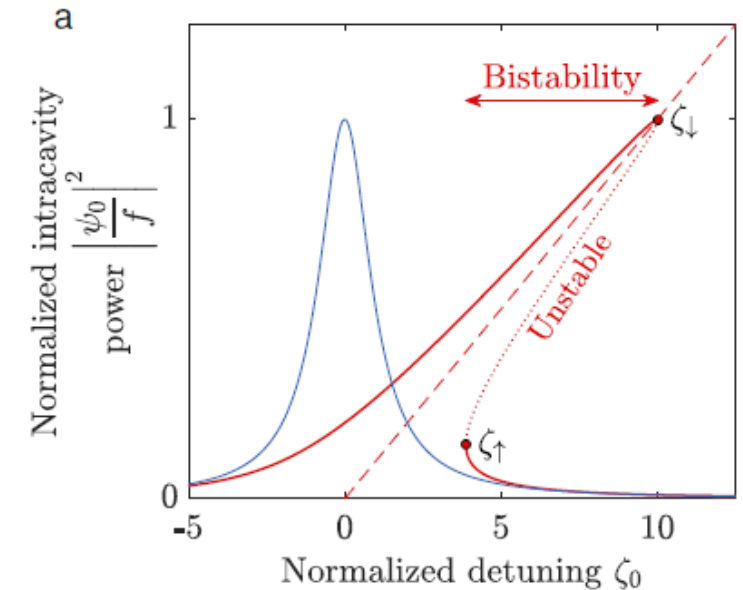
$$\beta = \frac{-D_2}{\kappa}$$

$$\zeta_0 = \frac{2\delta\omega}{\kappa}$$

Nonlinear response

- Can identify different regimes depending on the pump power and detuning:

- Continuous wave – *CW*
 - Tilting of the cavity resonance, creates bistable region
- Modulation instability – *MI*
 - Formation and growth of sidebands (periodic patterns).
 - Can be stable or chaotic
- Dissipative Kerr solitons – *DKS*
 - Optical pulses
 - Rely on double balance dispersion/nonlinearity and loss/gain.



Bistable region:

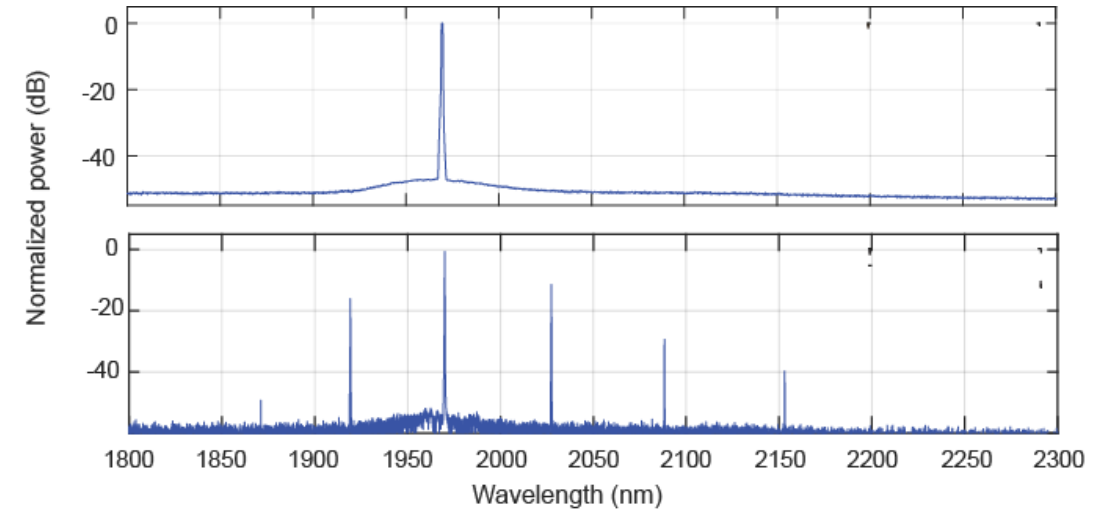
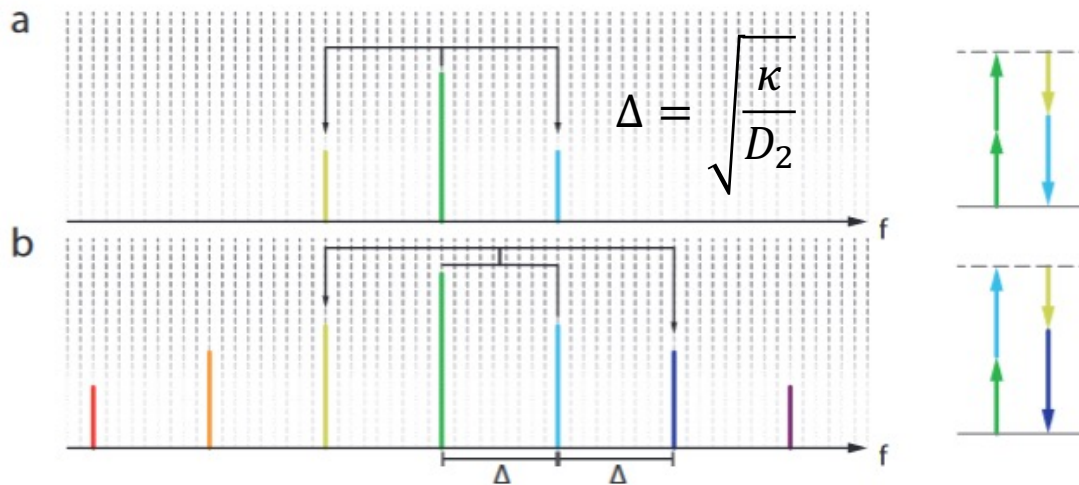
$$\zeta_0 \in [\zeta_{\uparrow}, \zeta_{\downarrow}] \text{ with } \zeta_{\uparrow} \approx 3(f^2/4)^{1/3}, \zeta_{\downarrow} \approx f^2$$

Frequency comb generation

- The homogenous CW region exhibits modulation instability (MI) when the intracavity energy is beyond a threshold. The minimum input pump power to reach MI is:

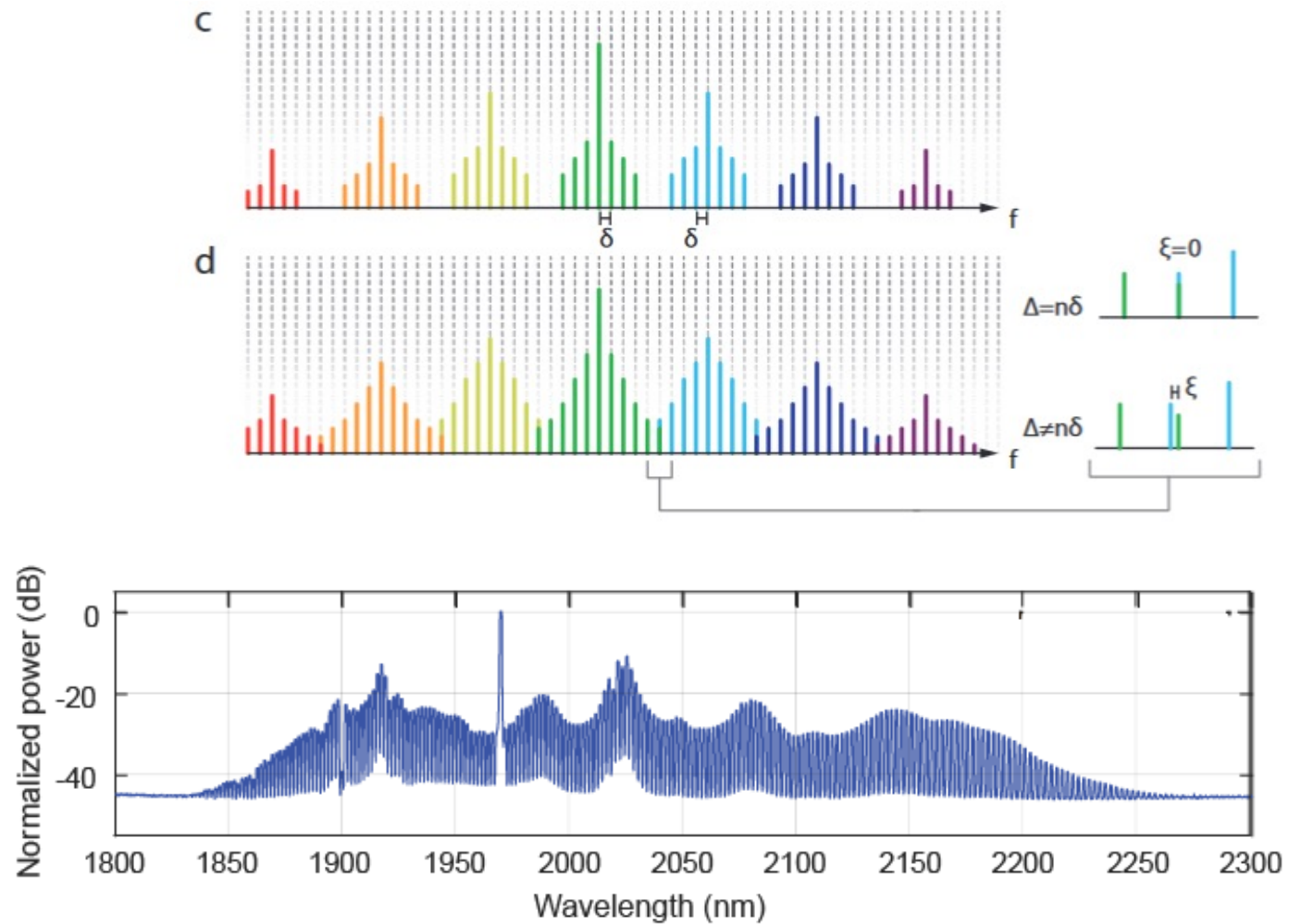
$$|\psi|^2 > 1 \Rightarrow f_{\text{thres}}^2 = 1 \quad P_{\text{thres}} = \frac{n_g^2}{8\omega_0 c_0} \frac{V_{\text{eff}}}{n_2} \frac{\kappa^2}{\eta}$$

- Depends on cavity properties : effective mode volume, nonlinear refractive index
 - Quadratic influence of the cavity linewidth (i.e. Q factor)
- CW solution becomes unstable. Formation of sidebands due to four wave mixing



MI combs

- Increasing the intracavity power leads to destabilization of the Turing pattern.

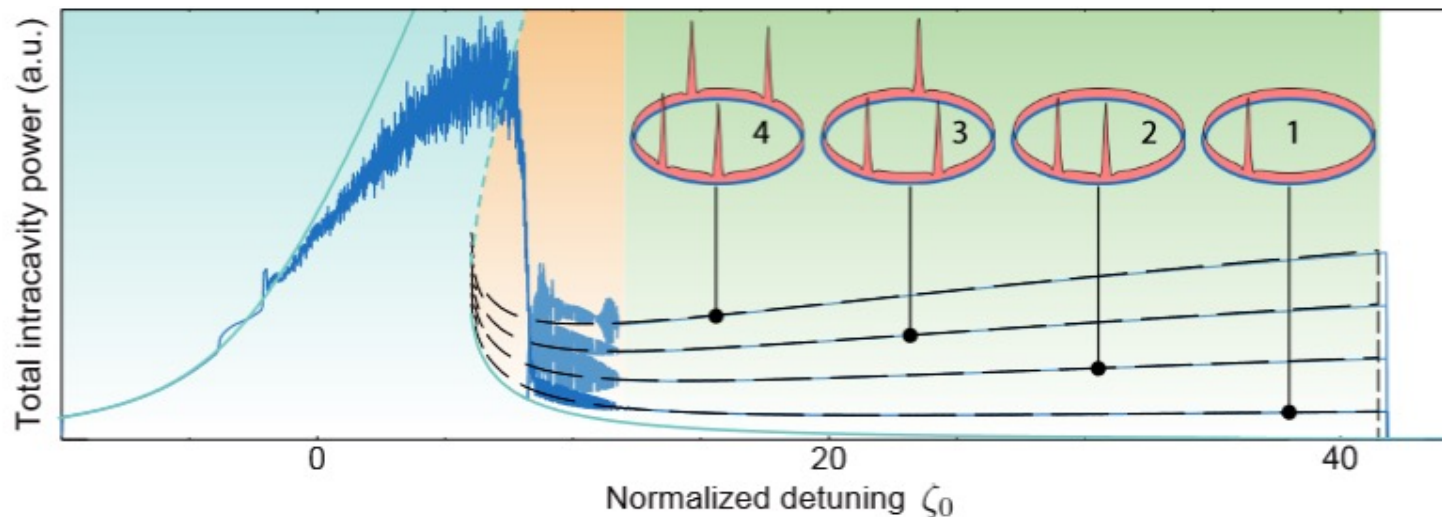


Dissipative Kerr solitons

- The analytical soliton solution to the LLE is not known.
 - Good approximation similar to the fundamental soliton solution of the NLSE
 - secant hyperbolic profile plus a continuous wave
 - The soliton existence range is approximately the bistable region ($\zeta_{\max} = \pi^2 f^2 / 8$)
- There can be N solitons in the cavity at various position ϕ_i :

$$\psi^{(N)}(t, \phi) = \psi_0 + \sqrt{2\zeta_0} e^{i\varphi_s} \sum_{k=1}^N \operatorname{sech} \left(\sqrt{\frac{\kappa \zeta_0}{D_2}} (\phi - \phi_k) \right)$$

$$N_{\max} \approx \sqrt{\frac{\kappa}{D_2}}$$

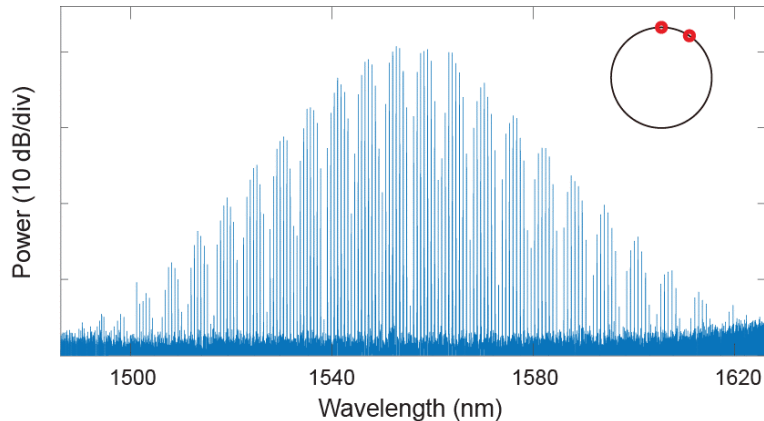


Multi solitons and soliton crystals

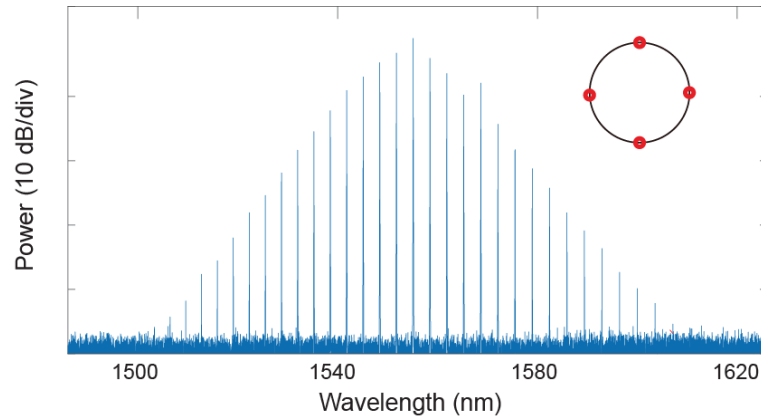
Soliton crystals: **tightly packed solitons** that fully occupy the resonator and exhibit collective temporal ordering, filling the angular domain of the resonator

Perfect soliton crystals: **Evenly distributed solitons** on the resonator circumference

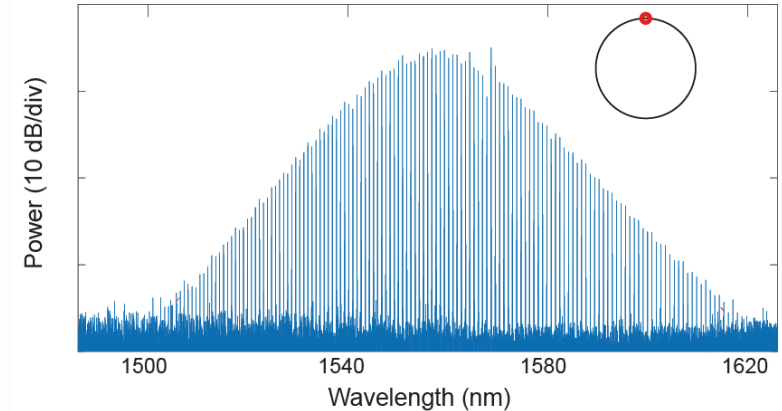
Two-soliton comb



4-PSC



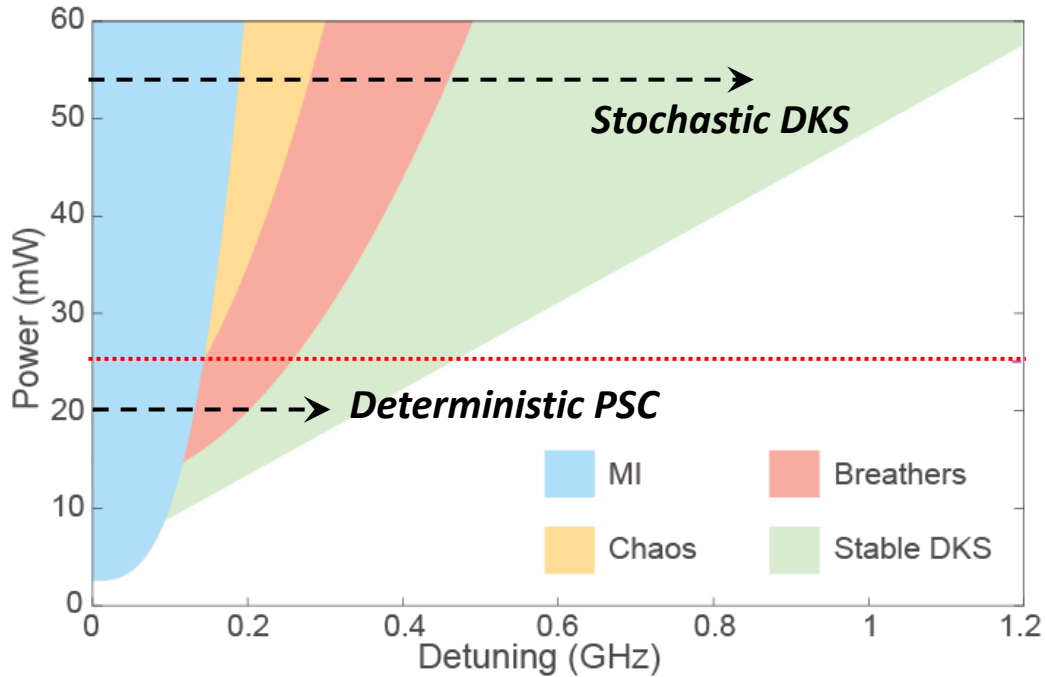
Single soliton



Formation of structures is linked to perturbation giving rise modulation on CW intracavity background leading to the formation of long-range bound states.

In microcavity: **avoided mode crossings** plays a significant role

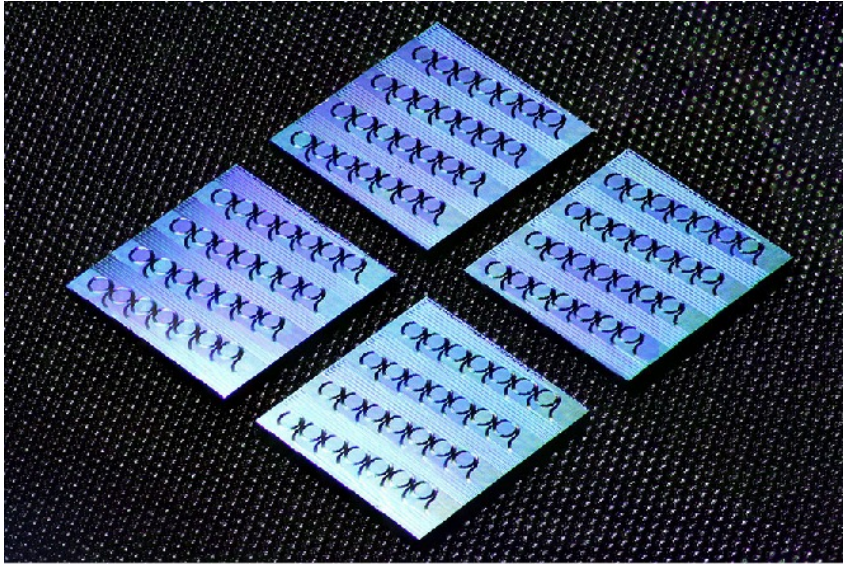
Versatile soliton generation



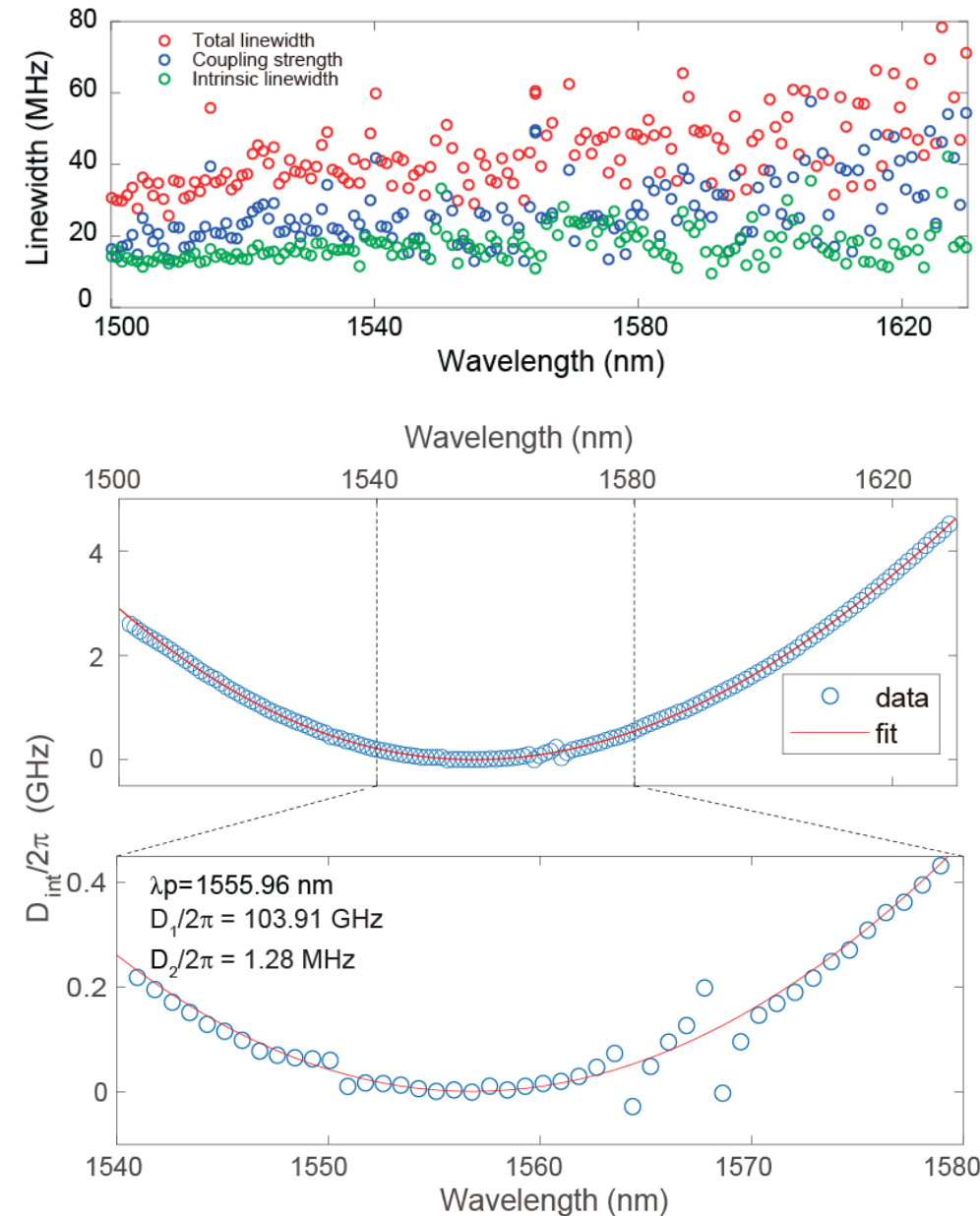
M. Karpov et al., Nature Physics 15 (2019)

- At higher pump power, dissipative Kerr solitons (DKS) with stochastic soliton number is generated.
- At lower pump power, equally-spaced perfect soliton crystals (PSC) are formed deterministically.

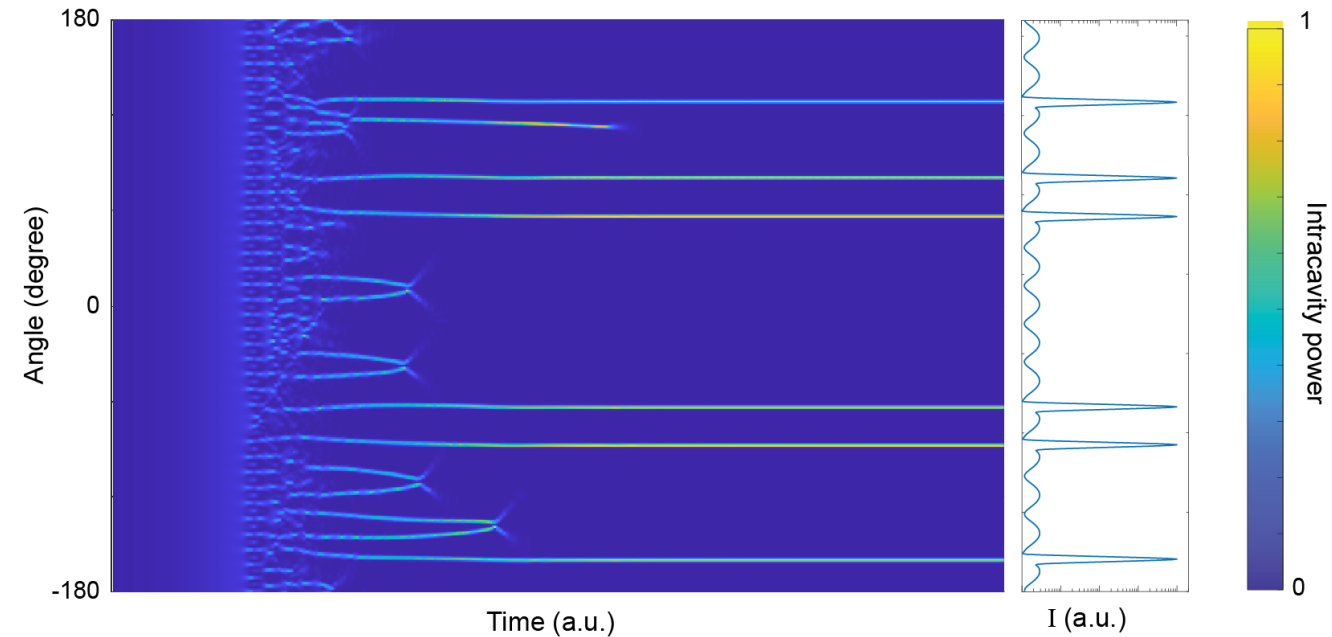
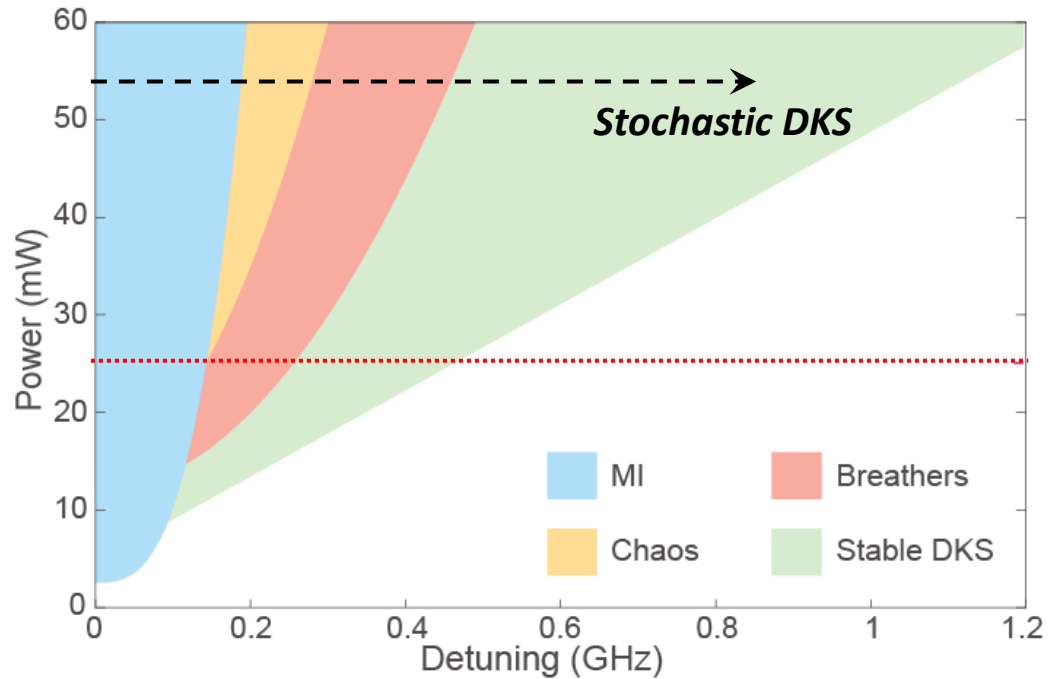
Experiment – *the resonator*



- Si_3N_4 microresonator, $217\text{ }\mu\text{m}$ radius
 - $Q > 10^7$
 - $\kappa_0/2\pi = \kappa_{\text{ex}}/2\pi = 20\text{ MHz}$
 - Pumped near 1555 nm
 - Anomalous dispersion, with major avoided mode crossings near the 15^{th} mode

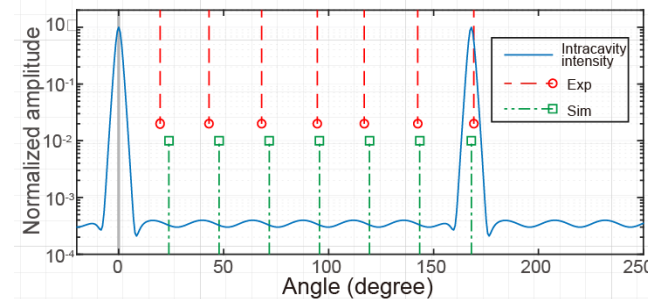
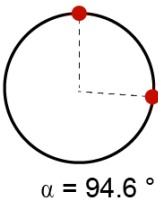
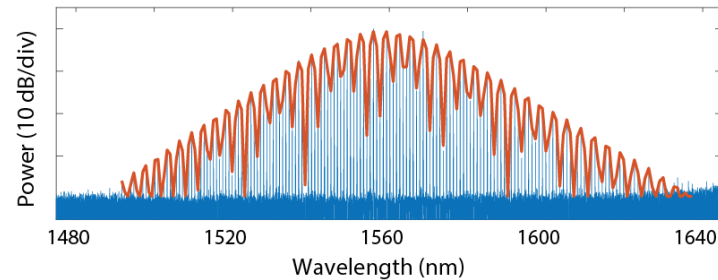
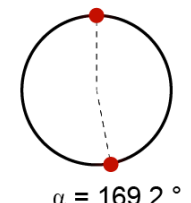
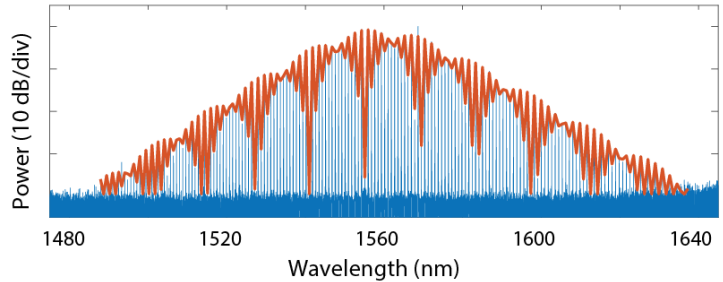
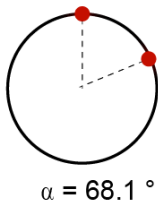
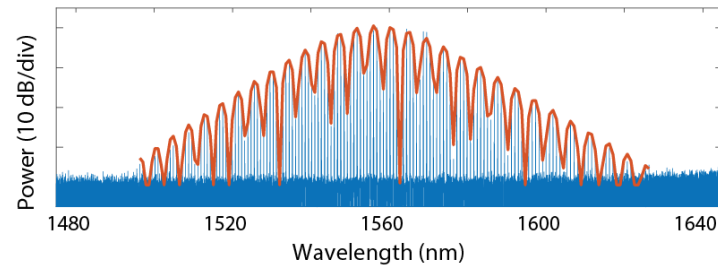
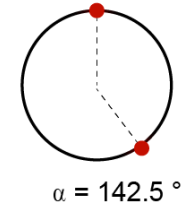
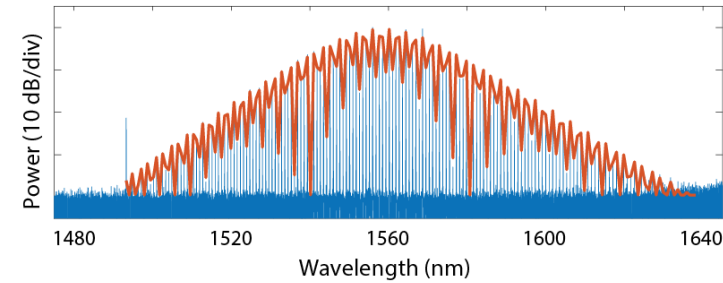
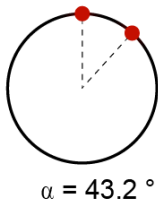
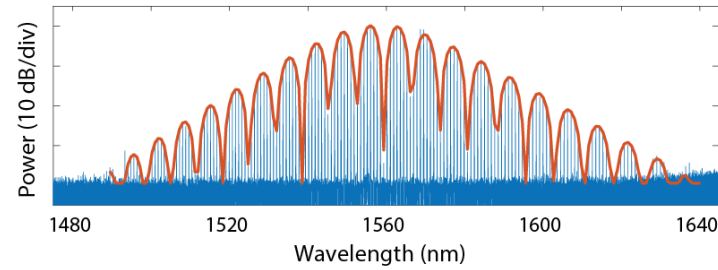
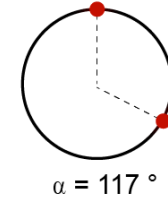
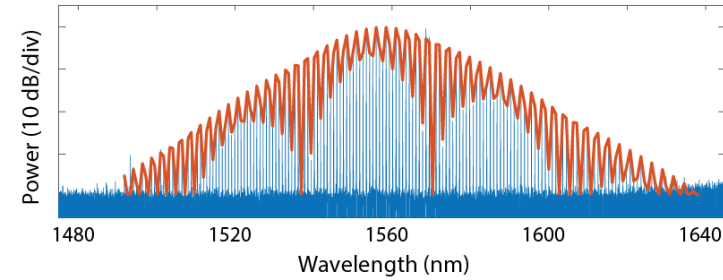
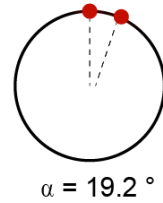
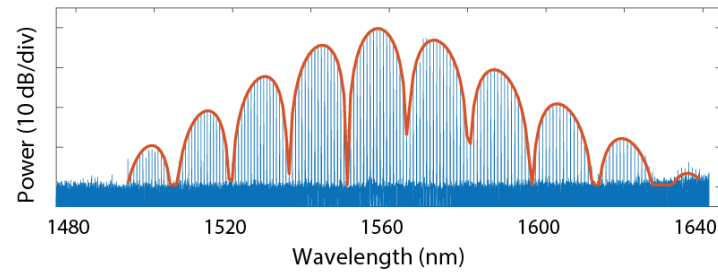


Attractor map – *above threshold*

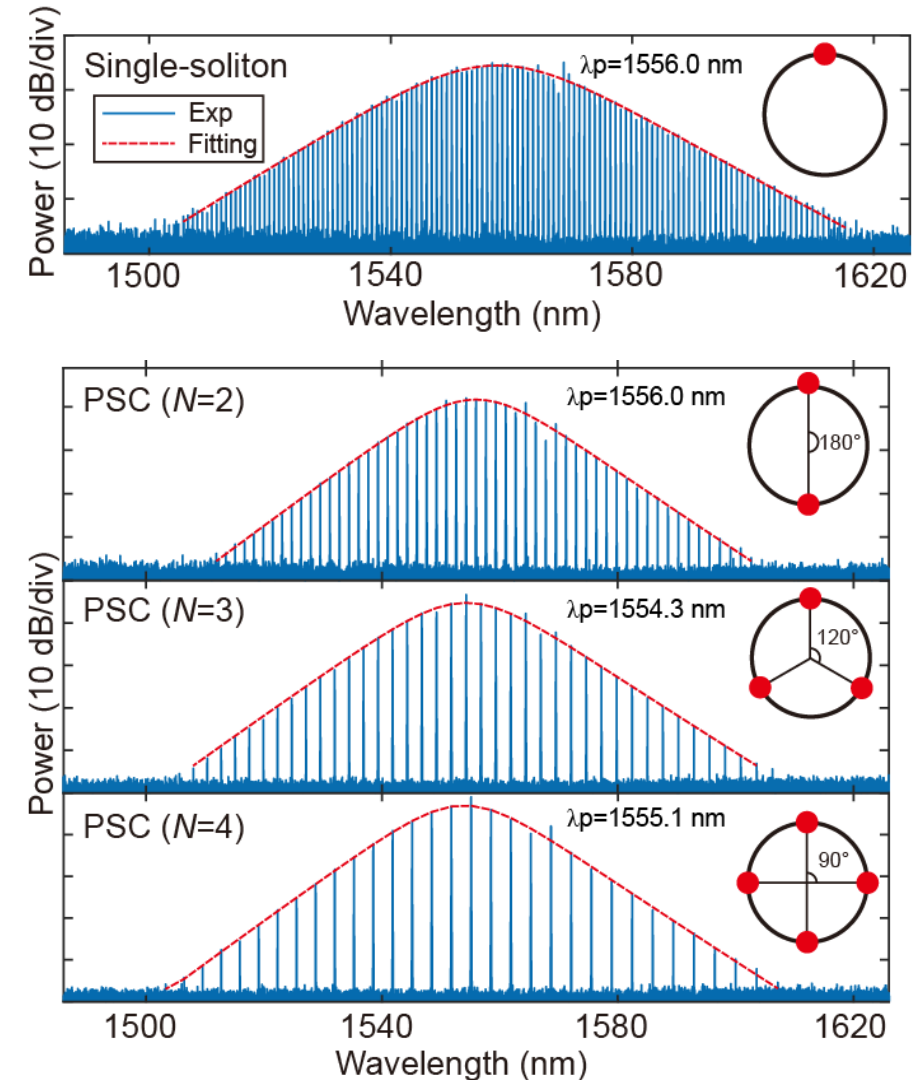
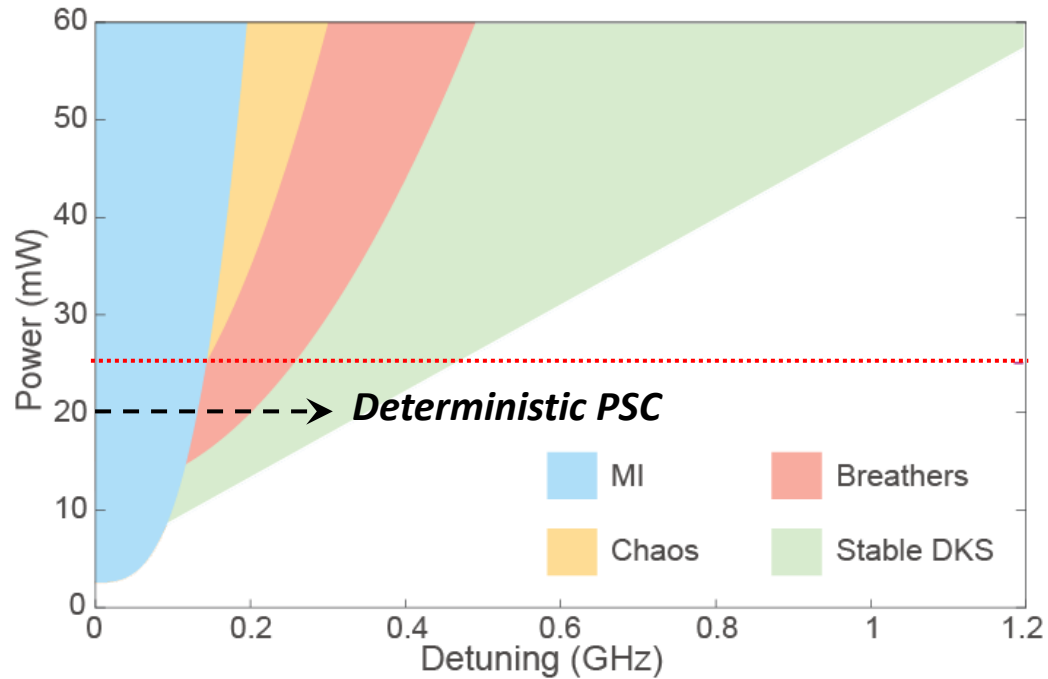


- Sweeping through chaos region clears some solitons and results in a **stochastic generation of soliton crystals**.
- The **relative position** of the solitons **should however be deterministic**, following the modulated CW background.

Two soliton microcombs - *experiments*



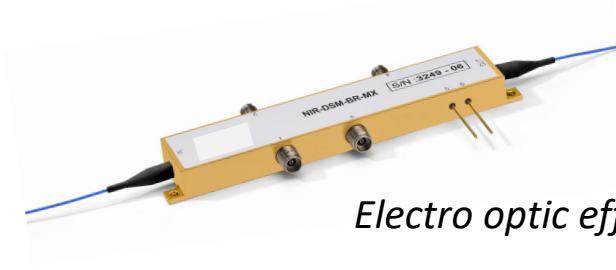
Perfect soliton crystals



By pumping different resonances at low power, single soliton and 3 different PSC (N=2, 3 and 4) could be deterministically generated with 100% probability

Challenges of integrated nonlinear optics

$\chi^{(2)}$ nonlinearities are key for several essential building blocks...

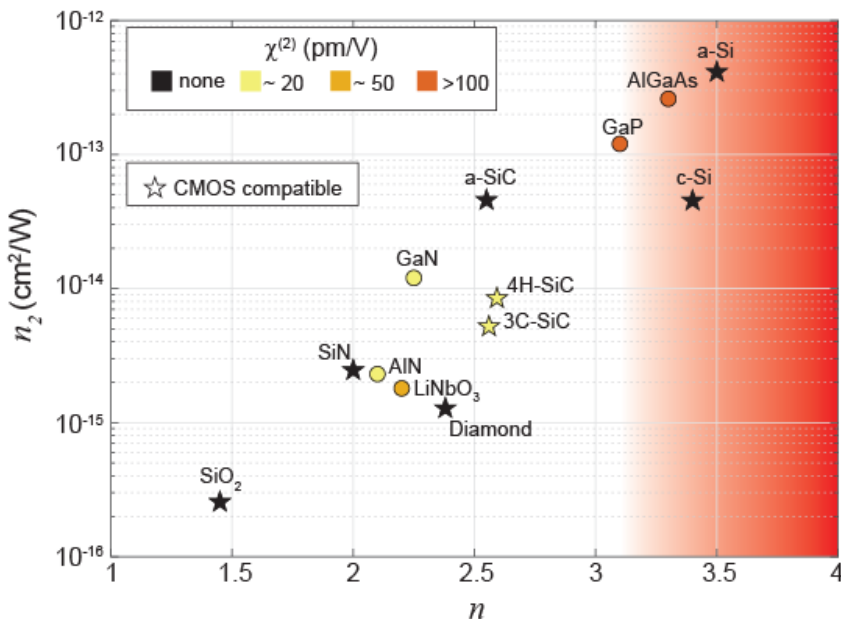


Electro optic effect



Three wave mixing

but the most mature photonic integrated platforms lack $\chi^{(2)}$ response



➔ *Effective $\chi^{(2)}$ in centrosymmetric platforms $\chi_{eff}^{(2)} = 3\chi^{(3)}E_{DC}$*

Si: E. Timurdogan, *et al.* **Nature Photonics** 11, pp 200–206 (2017)

SiN: X. Lu *et al.*, **Nature Photonics** 15, (2021), E. Nitiss *et al.*, **Nature Photonics** 16, (2022)
B. Zabelich *et al.*, **APL** 9, 1 (2024)

➔ *Emerging platforms with combined nonlinearities*

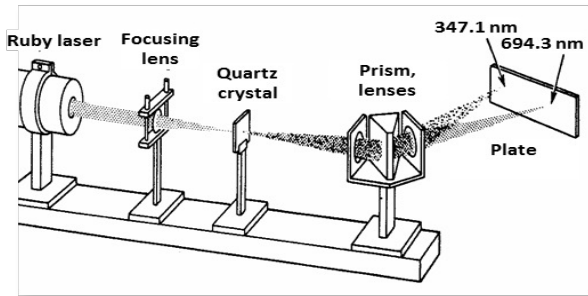
TFLN: A. Boes *et al.*, **Laser & Photonics Reviews** 12(4), (2018), Y. Qi, Y. Li, **Nanophotonics** 9(6), (2020), G. Chen *et al.*, **Advanced Photonics** 4(3), (2022)

AlN: X. Liu *et al.*, **Optica** 5(10), (2018).

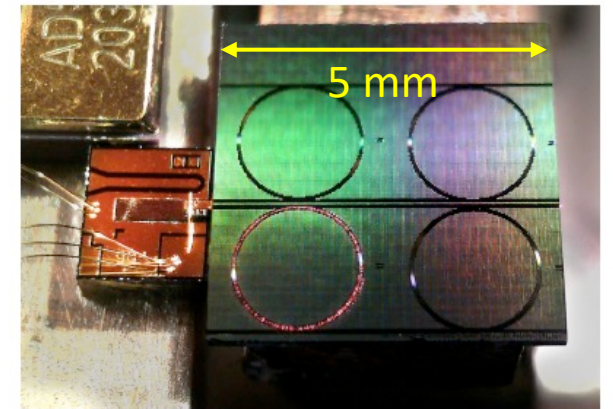
SiC: M. Guidry *et al.*, **Nat Photonics** 16, (2022).

Conclusion

Combination of **nanofabrication advancements**, **material** evolution and **novel designs leveraging 3D engineering** make integrated nonlinear photonic quickly evolve and applicable to several application domains



P. A. Franken, *et al.* **Phys. Rev. Lett.** 7, 118 (1961)



M. Clementi, *et al.*, **Light: Science & Applications.** 12 (2023)

Thank you !